An Interpretation of Fourier Series

Fourier Analysis is a name sometimes used to denote the decomposition of a function $f(t)$ into the sum of its sinusoidal harmonics, which are also called its Fourier components. Fourier Synthesis is a name used to denote the building up of a function $f(t)$ by adding up its successive Fourier components; that is, the reconstruction of a function $f(t)$ from its Fourier components.

The are electronic devices which can perform both types of operations, called Fourier analyzers and Fourier synthesizers. The ear and the brain also function as, respectively, a Fourier analyzer and a Fourier synthesizer! We describe this briefly (in very general terms).

The input to the ear is a time-varying pressure wave-form $f(t)$. The inner ear has an array of about twenty thousand hair-like cells, each of which resonates at a different frequency. Each of the individual Fourier components of $f(t)$ stimulates a different one of these "hair" cells. Thus this array of cells acts all together as a Fourier *analyzer*. Each hair cell which gets selected by a component of $f(t)$ to be driven into motion then stimulates an attached nerve which sends a signal to the brain. The brain (being the smart and capable device it is) then somehow combines or *synthesizes* these individual received Fourier components and produces a reconstructed approximation of the function $f(t)$ as their sum. This reconstructed $f(t)$-pattern in the brain is then what we experience as the sound corresponding to the incoming pressure-wave signal $f(t)$.

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