

## Part II Problems

**Problem 1:** [Periodic solutions] Let  $g(t)$  be the function which is periodic of period  $2\pi$ , and such that  $g(t) = t$  for  $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$  and  $g(t) = \pi - t$  for  $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$ .

(a) Find a periodic solution to  $\ddot{x} + \omega_0^2 x = g(t)$  (if there is one).

(b) For what (positive) values of  $\omega_0$  are there no periodic solution?

(c) Write  $\omega_r$  for the smallest number you found in (b). For  $\omega_0$  just less than  $\omega_r$ , what is the solution like, approximately? How about for  $\omega_0$  just larger than  $\omega_r$ ?

(d) For what values of  $\omega_0$  are there more than one periodic solution?

(e) For the values of  $\omega_0$  found in (d), are *all* solutions to  $\ddot{x} + \omega_0^2 x = g(t)$  periodic?

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