

18.03SC Practice Problems 27

Laplace transform

Rules for the Laplace transform

Definition: $\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$ for $\text{Re}(s) \gg 0$.

Linearity: $\mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s)$.

\mathcal{L}^{-1} : $F(s)$ essentially determines $f(t)$ for $t > 0$.

s -shift rule: $\mathcal{L}[e^{rt}f(t)] = F(s - r)$.

s -derivative rule: $\mathcal{L}[tf(t)] = -F'(s)$.

t -derivative rule: $\mathcal{L}[f'(t)] = sF(s) - f(0^-)$.

Formulas for the Laplace transform

$$\mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[\delta(t - a)] = e^{-as}$$

$$\mathcal{L}[e^{rt}] = \frac{1}{s - r}, \quad \mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}[\cos(\omega t)] = \frac{s}{s^2 + \omega^2}, \quad \mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2 + \omega^2}$$

1. Use the rules and formulas to find the Laplace transform of $e^{-t}(t^2 + 1)$.

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