18.03SC Practice Problems 13

Undetermined coefficients

Solution suggestions:

1. *Find the polynomial solution of* $\ddot{x} - x = t^2 + t + 1$.

Since no term in the right hand side satisfies the associated homogeneous equation (the constant term in $p(D) = D^2 - 1$ is nonzero), we can use the method of undetermined coefficients to solve by guessing a quadratic solution $x(t) = at^2 + bt + c$ and determining the a, b and c that work by substituting the guessed general form for $x(t)$ into the differential equation and comparing both sides.

You can do this yourself by whatever method you feel most comfortable with. A graphical technique for carrying out the work is to make a table like the one below. Write out the multipliers of the system along the left, fill out the table from the bottom up, compute the right-hand side in the bottom row, and then read off the conditions that the coefficients of the guessed solution must satisfy.

$$\begin{array}{rcr}
1 \mid \ddot{x} = & & 2a \\
0 \mid \dot{x} = & & 2at + b \\
-1 \mid x = & & at^2 + bt + c \\
---- & & -------- \\
t^2 + t + 1 \quad = & & (-a)t^2 + (-b)t + (2a - c)
\end{array}$$

Here the conditions we get are the three equations $-a = 1$, $-b = 1$ and $2a - c = 1$. Solving these equations simultaneously, we obtain that $a = -1$, $b = -1$ and $c = -3$.

So the polynomial solution of the equation is

$$x(t) = -t^2 - t - 3.$$

As a sanity check, you can make sure this is indeed a solution by plugging it in to the original equation.