

## Time Invariance

In the case of constant coefficient operators  $p(D)$ , there is an important and useful relationship between solutions to  $p(D)x = q(t)$  for input signals  $q(t)$  which start at different times  $t$ . The following result shows why these operators are called “Linear Time Invariant” (or LTI).

**Translation invariance.** If  $p(D)$  is a constant-coefficient differential operator and  $p(D)x = q(t)$ , then  $p(D)y = q(t - c)$ , where  $y(t) = x(t - c)$ .

This is the “time invariance” of  $p(D)$ . Here is an example of its use.

**Example.** Suppose that we know that  $x_p(t) = \sqrt{2} \sin(t/2 - \pi/4)$  is a solution to the DE

$$2\ddot{x} + \dot{x} + x = \sin(t/2) \quad (1)$$

Find a solution  $y_p$  to

$$2\ddot{x} + \dot{x} + x = \sin(t/2 - \pi/3) \quad (2)$$

**Solution.** By translation-invariance, we have immediately that

$$y_p = \sqrt{2} \sin(t/2 - \pi/4 - \pi/3) = \sqrt{2} \sin(t/2 - 7\pi/12).$$

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18.03SC Differential Equations  
Fall 2011

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