Linear Differential Operators With Constant Coefficients

The general linear ODE of order n for a function $y = y(t)$ can be written as

$$y^{(n)} + p_1(t)y^{(n-1)} + \ldots + p_n(t)y \;=\; q(t). \tag{1}$$

From now on we will consider only the case where (1) has constant coefficients. This type of ODE can be written as

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y \;=\; q(t) \tag{2}$$

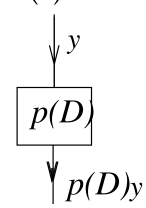
or, as we have seen, much more compactly using the differentiation operator $D = \dfrac{d}{dt}$:

$$p(D)\, y \;=\; q(t)\,,$$

where

$$p(D) = D^n + a_1 D^{n-1} + \ldots + a_n. \tag{3}$$

We call $p(D)$ a **polynomial differential operator with constant coefficients**. We think of the formal polynomial $p(D)$ as operating on a function $y(t)$, converting it into another function; it is like a black box, in which the function $y(t)$ goes in, and $p(D)y$ (i.e., the left side of (2)) comes out.



The reason for introducing the polynomial operator $p(D)$ is that this allows us to use polynomial algebra to simplify, streamline and extend our calculations for solving CC DE's. Throughout this session we use the notation of equation (4):

$$p(D) = D^n + a_1 D^{n-1} + \ldots + a_n\,, \qquad a_i \text{ constants.} \tag{4}$$

MIT OpenCourseWare
http://ocw.mit.edu

18.03SC Differential Equations
Fall 2011

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.