Operators

Operators are to functions as functions are to numbers. An operator takes a function, does something to it, and returns this modified function. There are lots of examples of operators around:

—The *shift-by-a operator* (where a is a number) takes as input a function $f(t)$ and gives as output the function $f(t-a)$. This operator shifts graphs to the right by a units.

—The *multiply-by-$h(t)$ operator* (where $h(t)$ is a function) multiplies by $h(t)$: it takes as input the function $f(t)$ and gives as output the function $h(t)f(t)$.

You can go on to invent many other operators. In this course the most important operator is:

—The *differentiation operator*, which carries a function $f(t)$ to its derivative $f'(t)$.

The differentiation operator is usually denoted by the letter D; so $Df(t)$ is the function $f'(t)$. D carries f to f'. For example, $Dt^3 = 3t^2$. This is usually read as "D *applied* to t^3."

The *identity operator* takes an input function $f(t)$ and returns the *same* function, $f(t)$; it does nothing, but it still gets a symbol, I: $If = f$.

Operators can be added and multiplied by numbers or more generally by functions. Thus $tD + 4I$ is the operator sending $f(t)$ to $tf'(t) + 4f(t)$.

The single most important concept associated with operators is that they can be *composed* with each other. Composition of two operators in a given order means that the two operators are applied to a function one after the other. For example, D^2, the second-derivative operator, means differentiation twice, sending $f(t)$ to $f''(t)$. It is in fact the composition of D with itself: $D^2 = D \cdot D$, so that $D^2 f = D(Df) = D(f') = f''$.