Part II Problems

Problem 1: [Series RLC circuits; amplitude and phase] Open the Mathlet `Series RLC Circuit`. Here we will focus entirely on the current response, so it will be clearer if the check boxes labelled V_R, V_L, V_C, are left unchecked. But click twice on the I box, to make a green curve appear in the graphing window, representing the current through any point in the circuit as a function of time.

The Mathlet uses the International System of Units, SI, formerly known as the mks (meter-kilogram-second) system. The equation

$$L\ddot{I} + R\dot{I} + (1/C)I = \dot{V}$$

is correct when:
the resistance R is measured in ohms, Ω,
the inductance L is measured in H, henries,
the capacitance C is measured in farads, F,
the voltage V is measured in volts, also denoted V,
the current I is measured in amperes, A.

The slider displays millihenries, mH (1 mH= 10^{-3} H) and microfarads, μF (1μF= 10^{-6} F), and milliseconds, ms (1 ms = 10^{-3} sec).

The Mathlet studies a sinusoidal input signal $V(t) = V_0 \sin(\omega t)$. Play around with the various sliders and watch the effect on the (blue) voltage curve and the (green) current curve.

(a) By experimenting, identify a few values of the system parameters R, L, C, V_0, ω, for which the current and the voltage are perfectly *in phase*. For example, if $L = 500$ mH and $\omega = 200$ radians/second, what values of R, C, and V_0 put I in phase with V?

(b) Now calculate the relationship between the system parameters which leads to I and V being in phase. Do your experiments align with your calculations?

(c) Set $R = 100\,\Omega$, $L = 1000$ mH, $C = 100\,\mu$F, $V_0 = 500$ V. Vary ω and watch the action. For what value of ω is the amplitude of $I(t)$ maximal? What is that amplitude (in amps)? What is the phase lag between the input signal, $V_0 \sin(\omega t)$, and the system response, $I(t)$, for that value of ω?

(d) Verify the three observations made in **(c)** computationally. You should be able to do this for general values of R, L, C, V_0.

Problem 2: AM Radio Tuning and LRC Circuits

An LRC circuit can be modeled using the same DE as in the previous problem. Specifically,

$$LI'' + RI' + \frac{1}{C}I = E'.$$

Where I = current in amps, L = inductance in henries, R = resistance in ohms, C = capacitance in farads and E = input EMF in volts. Often the important output is the voltage drop V_R across the resistor. Ohm's law tells us $V_R = RI$. This gives us the DE

$$LV_R'' + RV_R' + \frac{1}{C}V_R = RE'.$$

(a) Assume $E = E_0 \cos(\omega t)$ and solve the DE for V_R in phase-amplitude form.

(b) Open the 'LRC Filter Applet'. This applet models an LRC circuit with input voltage a superposition of sine waves. Play with the applet –be sure to learn how to vary ω_1 and ω_2 by dragging the dots on the amplitude plot.

Describe what happens to the amplitude response plot as L, R and C are varied.

(c) An LRC circuit can be used as part of a simple AM radio tuner. In an AM radio broadcast the signal is given by $a\cos(\omega t)$ where ω is the 'carrier' frequency (between 530 and 1600 khz). To really carry information the amplitude a must vary –this is the amplitude modulation– but, we will ignore this right here.

The range of values for this simple variable capacitor AM radio tuner is $L \approx .5$ microhenries, R is the resistance in the wire (very small) and C is between .02 and .2 microfarads. To keep things simple we will use different ranges however the idea is the same.

In the LRC Filter applet set $\omega_1 = 1$ and $w_2 = 4$ (set them as close as you can on your system). Set the input amplitudes a and b to 1. Find settings for L, R and C so that the output filters out the ω_2 part of the signal i.e. the output looks (a lot) like a sine wave of frequency ω_1. Give your values for L, R and C.

How does the quality of the filter change as you vary R?

(d) An antenna on a radio picks up electomagnetic signals from all frequencies. It responds by outputing a signal consisting of voltages at each of these frequencies. This signal is used as input to a tuner circuit.

Using the applet, set $L = 1$, $R = .5$. Now, vary C and then explain why a variable capacitor circuit could be used as an AM radio tuner.

(e) Show that the natural frequency (undamped, unforced resonant frequency) of the system is $\omega_0 = 1/\sqrt{LC}$. Show that even with damping, i.e., $R > 0$, ω_0 is always the practical resonant frequency. (Hint: this can be done without calculus by writing $A(\omega)$ in the proper way.)

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