Sinusoidal Functions

1. Definitions

A **sinusoidal function** (or **sinusoidal oscillation** or **sinusoidal signal**) is one that can be wrtten in the form

$$f(t) = A\cos(\omega t - \phi). \tag{1}$$

The function $f(t)$ is a cosine function which has been **amplified** by A, **shifted** by ϕ/ω, and **compressed** by ω.

- $A > 0$ is its **amplitude**: how high the graph of $f(t)$ rises above the t-axis at its maximum values;
- ϕ is its **phase lag**: the value of ωt for which the graph has its maximum (if $\phi = 0$, the graph has the position of $\cos(\omega t)$; if $\phi = \pi/2$, it has the position of $\sin(\omega t)$);
- $\tau = \phi/\omega$ is its **time delay** or **time lag**: how far along the t-axis the graph of $\cos(\omega t)$ has been shifted to make the graph of (1); (to see this, write $A\cos(\omega t - \phi) = A\cos(\omega(t - \phi/\omega)))$
- ω is its **angular frequency**: the number of complete oscillations $f(t)$ makes in a time interval of length 2π; that is, the number of radians per unit time;
- $\nu = \omega/2\pi$ is the **frequency** of $f(t)$: the number of complete oscillations the graph makes in a time interval of length 1; that is, the number of cycles per unit time;
- $P = 2\pi/\omega = 1/\nu$ is its **period**, the t-interval required for one complete oscillation.

One can also write (1) using the time lag $\tau = \phi/\omega$

$$f(t) = A\cos\left(\omega(t - \tau)\right).$$

2. Discussion

Here are the instructions for building the graph of (1) from the graph of $\cos(t)$. First *scale*, or vertically stretch, $\cos(t)$ by a factor of A; then *shift* the result to the right by ϕ units (if $\phi < 0$ the shift will actually be to the left); and finally *scale* it horizontally by a factor of $1/\omega$.

In the figure below the dotted curve is $\cos(t)$ and the solid curve is $2.5\cos(\pi t - \pi/2)$. The solid curve has

$$A = 2.5, \quad \omega = \pi, \quad \phi = \pi/2, \quad \tau = 1/2.$$

Vertically, the solid curve is 2.5 times the dotted one. Horizontally, the solid curve it $1/\pi$ times the dotted one. (The dotted curve takes 2π units of time to go through one cycle and the solid curve takes only 2 units of time.) The solid curve hits its first maximum at $t = 1/2$, i.e. at the $t = \tau$, the time lag.

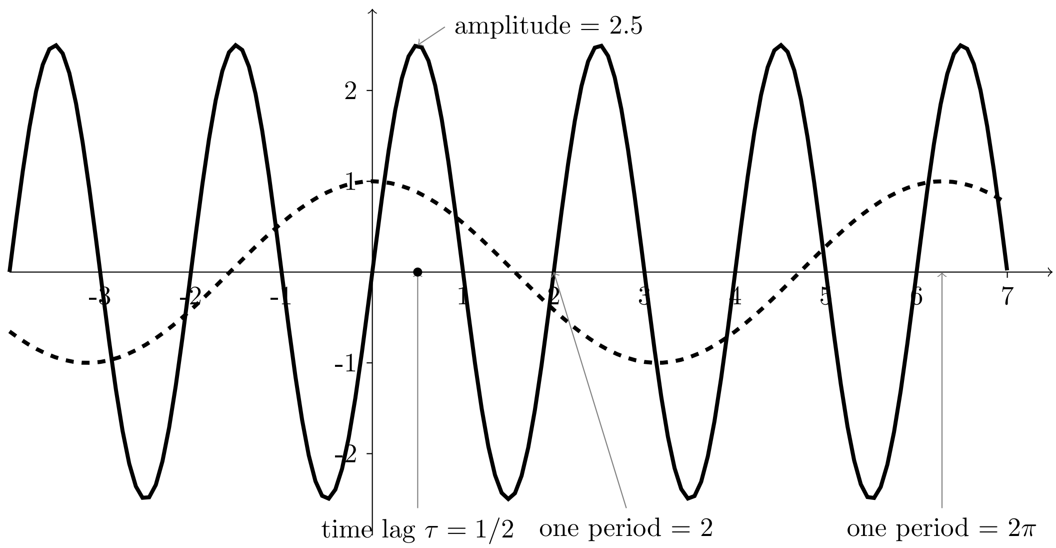

Fig. 1. Features of the graph of a sinusoid.