# 18.03SC Practice Problems 7

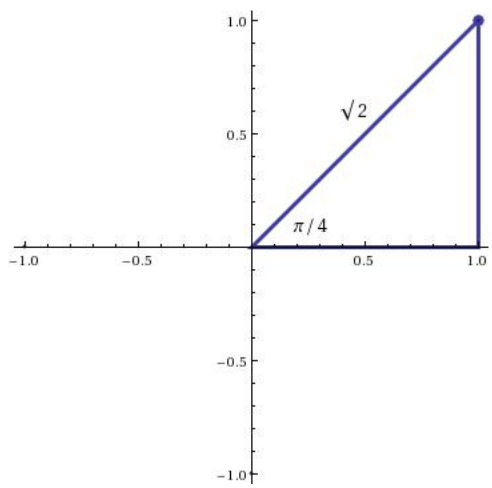
## Sinusoids

## Solution Suggestions

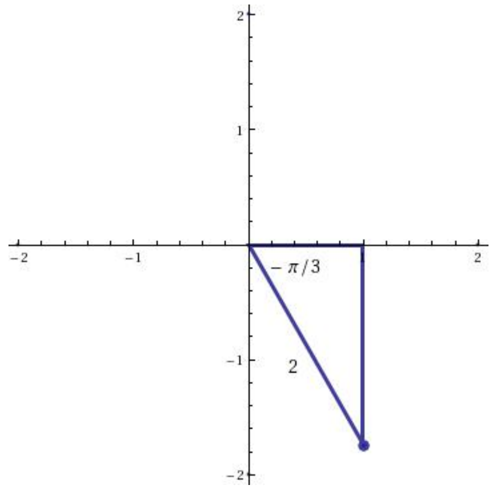
**1.** *Write each of the following functions (of t) in the form $A\cos(\omega t - \phi)$. In each case, begin by drawing a right triangle with sides a and b.*
*(a)* $\cos(2t) + \sin(2t)$.
*(b)* $\cos(\pi t) - \sqrt{3}\sin(\pi t)$.
*(c)* $\mathrm{Re}\dfrac{e^{it}}{2+2i}$.
Recall the geometric derivation of the general sinusoid formula

$$a\cos(\omega t) + b\sin(\omega t) = \overline{(a,b)} \cdot \overrightarrow{(\cos(\omega t), \sin(\omega t))} = \sqrt{a^2+b^2}\cos(\omega t - \phi),$$

where $\phi$ is the angle of the first vector. This is where the triangle comes in - we will draw the vector of coefficients to determine its magnitude and direction. (Equivalently, we are determining the polar coordinates of the complex number $a + bi$.)
**(a)** $\cos(2t) + \sin(2t)$: $(a,b) = (1,1)$.




Here, the right triangle has hypotenuse $\sqrt{1+1} = \sqrt{2}$, so $A = \sqrt{2}$. Both summands have angular frequency 2, so $\omega = 2$. $\phi$ is the angle of the triangle, which is $\pi/4$, so $\cos(2t) + \sin(2t) = \sqrt{2}\cos(2t - \pi/4)$.
**(b)** $\cos(\pi t) - \sqrt{3}\sin(\pi t) : (a,b) = (1, -\sqrt{3})$.




This right triangle has hypotenuse $\sqrt{1^2 + (-\sqrt{3})^2} = 2$ and angle $-\pi/3$. So $\cos(\pi t) - \sqrt{3}\sin(\pi t) = 2\cos(\pi t - (-\pi/3)) = 2\cos(\pi t + \pi/3)$.

**(c)** $e^{it} = \cos(t) + i\sin(t)$ and $\frac{1}{2+2i} = \frac{1}{2+2i} \cdot \frac{2-2i}{2-2i} = \frac{2-2i}{2^2+2^2} = \frac{1-i}{4}$. Multiply out and take the real part of the product to obtain $\mathrm{Re}\frac{e^{it}}{2+2i} = \frac{1}{4}\cos(t) + \frac{1}{4}\sin(t)$.

For $\frac{1}{4}\cos(t) + \frac{1}{4}\sin(t)$, $(a, b) = (\frac{1}{4}, \frac{1}{4})$, which gives the same triangle as in (a), except scaled by $1/4$.

So $\mathrm{Re}\frac{e^{it}}{2+2i} = \frac{1}{4}\cos(t) + \frac{1}{4}\sin(t) = \frac{\sqrt{2}}{4}\cos(t - \pi/4)$.