Part I Problems and Solutions

Problem 1: *Write each of the following functions* $f(t)$ *in the form* $A\cos(\omega t - \phi)$*. In each case, begin by drawing a right triangle.*

a) $2\cos(3t) + 2\sin(3t)$

b) $\sqrt{3}\cos(\pi t) - \sin(\pi t)$

c) $\cos(t - \frac{\pi}{8}) + \sin(t - \frac{\pi}{8})$

Solution: a) Here, our right triangle has hypotenuse $2\sqrt{2}$, so $A = 2\sqrt{2}$. Both summands have circular frequency 3, so $\omega = 3$. ϕ is the argument of the hypotenuse, which is $\pi/4$, so $f(t) = 2\sqrt{2}\cos(3t - \pi/4)$.

b) The right triangle has hypotenuse of length $\sqrt{(\sqrt{3})^2 + (-1)^2)} = 2$. The circular frequency of both summands is π, so $\omega = \pi$. The argument of the hypotenuse is $-\pi/6$, so $f(t) = 2\cos(\pi t + \pi/6)$.

c) Similar to (a), with $3t$ replaced by $t - \pi/8$:

$$f(t) = \sqrt{2}\cos(t - \pi/8 - \pi/4) = \sqrt{2}\cos(t - \tfrac{3\pi}{8}).$$

Problem 2: *Find* $\int e^{2x}\sin x\,dx$ *by using complex exponentials.*

Solution:

$$\begin{aligned}
e^{(2+i)x} &= e^{2x}(\cos x + i\sin x) \\
e^{2x}\sin x &= \operatorname{Im} e^{(2+i)x} \\
\int e^{(2+i)x}dx &= \frac{1}{2+i}e^{(2+i)x} \\
&= \frac{2-i}{5}\left(e^{2x}\cos x + ie^{2x}\sin x\right)
\end{aligned}$$

We want just the imaginary part; multiplying out and collecting the coefficient of i then gives

$$\int e^{2x}\sin x\,dx = e^{2x}\left(\frac{2}{5}\sin x - \frac{1}{5}\cos x\right)$$