

### 18.03SC Practice Problems 3

#### Euler's method

#### Solution Suggestions

1. Use Euler's method to estimate the value at  $x = 1.5$  of the solution of  $\frac{dy}{dx} = y' = F(x, y) = y^2 - x^2$  for which  $y(0) = -1$ . Use step size  $h = 0.5$ . Recall the notation  $x_0 = 0, y_0 = -1, x_{n+1} = x_n + h, y_{n+1} = y_n + m_n h, m_n = F(x_n, y_n)$ . Make a table with columns  $n, x_n, y_n, m_n, m_n h$ . Draw the Euler polygon.

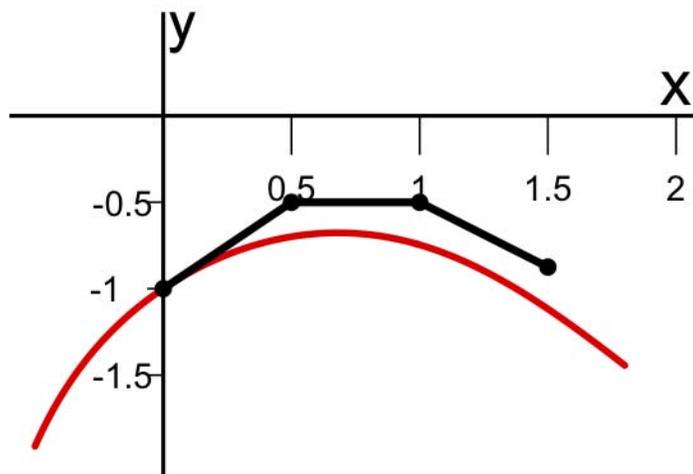
We are estimating  $y(1.5)$  using Euler's method with step size 0.5. This takes  $\frac{1.5-0}{0.5} = 3$  steps, as outlined in the following table.

$n$	$x_n$	$y_n$	$m_n$	$m_n h$
0	0	-1	1	0.5
1	0.5	-0.5	0	0
2	1.0	-0.5	-0.75	-0.375
3	1.5	-0.875		

Thus, Euler's method gives the estimate

$$y(1.5) \approx y_3 = -0.875.$$

The corresponding Euler polygon for this estimation is



Euler polygon and actual integral curve for Question 1.

2. Is the estimate found in Question 1 likely to be too large or too small?

It is likely to be too large. One way to see this is to use the second derivative test to check the concavity of solutions around the initial point. That is, find  $\frac{d^2y}{dx^2} = y''$  by taking the derivative of the differential equation

$$y'' = \frac{d}{dx}(F(x, y)) = \frac{d}{dx}(y^2 - x^2) = 2yy' - 2x,$$

and evaluate the second derivative at the initial point  $(x_0, y_0)$  to get  $y''|_{(0,-1)} = 2(-1)(1) - 2(0) = -2 < 0$ . This means the solution that goes through the initial point is concave down. The tangent to a concave down function lies above the function in a small neighborhood, so the Euler estimate for one step is likely to overshoot. Running the same check for the next two endpoints shows that the second derivative is negative at each endpoint of the Euler polygon, so each of the three steps is likely to overshoot. This suggests the estimate found is likely to be greater than the actual value of our solution  $y = y(x)$  at  $x = 1.5$ .

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