

Part II Problems

Problem 1: [Euler's method] **(a)** Write y for the solution to $y' = 2x$ with $y(0) = 0$. What is $y(1)$? What is the Euler approximation for $y(1)$, using 2 equal steps? 3 equal steps? What about n steps, where n can now be any natural number? (It will be useful to know that $1 + 2 + \cdots + (n - 1) = n(n - 1)/2$.) As $n \rightarrow \infty$, these approximations should converge to $y(1)$. Do they?

(b) In the text and in class it was claimed that for small h , Euler's method for stepsize h has an error which is at most proportional to h . The n -step approximation for $y(1)$ has $h = 1/n$. What is the exact value of the difference between $y(1)$ and the n -step Euler approximation? Does this conform to the prediction?

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