

## Existence and Uniqueness Theorem for ODE's

The following is a key theorem of the theory of ODEs; it has immediate (and crucial) consequences for sketching integral curves.

**Theorem.** Existence and Uniqueness Theorem for ODEs

For any  $(a, b)$  in the region where  $f$  is defined,  $y' = f(x, y)$  has exactly one solution such that  $y(a) = b$ .

**Remark 1.** The solution  $y(x)$  needn't be defined for all  $x$ : it may only exist for  $x$  near  $a$ .

**Remark 2.** You actually have to put some technical conditions on  $f$ . The most important one is that the theorem can fail if  $f(x, y)$  is not continuous at  $(a, b)$ . That is, there might be many solution curves or no solution curves that pass through the point.

By the existence part of the theorem, there is an integral curve through any point where  $f(x, y)$  is defined. Now suppose you have two integral curves through the same point. By the uniqueness part of the theorem, they must agree:

**Theorem.** Intersection Principle

Integral curves cannot intersect; every point lies on exactly one integral curve.

This principle is key in sketching integral curves on a direction field: you should never draw two that intersect, i.e. they can't cross or even touch at a point. We will see plenty of illustrations of the principle using the Isoclines applet.

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18.03SC Differential Equations  
Fall 2011

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