

Terminology: Systems and Signals

1. Systems, Signals, Input and Output

In the bank example we modeled the amount in a bank account by:

$$\dot{x} - rx = q(t).$$

Notice that the right-hand side does not depend on x . The left-hand side represents what happens at the bank. The right-hand side represents what comes into the bank. We will borrow the language of engineering to describe the pieces of our model.

The left-hand side represents the **system** (in this case, the bank). The right-hand side represents an outside influence on the system. We call it the **input signal** or sometimes simply the **input**.

In general, a signal is a function of t . Here it is the rate of savings (positive or negative). The system *responds* to the input signal and yields the function $x(t)$, which we call the **output signal** or **system response**. It is the current amount of money in the account.

Solving the ODE means finding the unknown x . In this new language it means finding the response of the system to the input signal $q(t)$.

It is important to understand that system and input are not mathematical terms. They are convenient, physically meaningful terms that we use to describe different parts of our ODE. What constitutes the input and output signals is a matter of interpretation of the physical setting, not a property of the equation itself.

Generally speaking it will be clear which is which, but in any example where it might be unclear we will specify the system, input and output.

2. Block Diagrams

Block diagrams are a useful tool that engineers use to represent systems. Here is a very simple block diagram which shows the input entering the system and the resulting output.

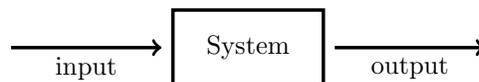


Fig. 1. Simple block diagram showing input, system and output.

A slightly more complicated diagram is needed to represent our bank example with an initial condition.

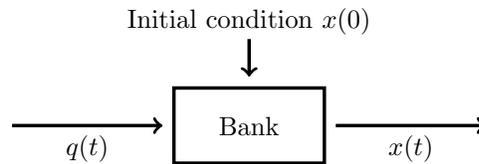


Fig. 2. This diagram shows the input signal $q(t)$ and initial condition $x(0)$ going into the system and the output signal $x(t)$ coming out.

3. RC Circuits

In this section we will look at the ODE for an RC circuit. The first reason for doing this is to introduce circuits into the course. They will be a fruitful source of interesting examples. When we study second order equations we will be able to model and study circuits that include inductors.

The second reason for using this example is to bring out an important point about what we call input. In the bank example, the input was simply the term on the right-hand side of the differential equation. In this example the right-hand side will be derived from the input, but will not be exactly the input.

This is not a course in electromagnetism or in circuits, but we will use examples from the subject. We will learn enough about circuits to use them as a source of interesting differential equations. One of the beautiful features of mathematical abstraction is that many systems can be modeled by the same DE. By understanding the DE we learn something about all the things modeled by it.

3.1. The Circuit

Suppose we have an electrical circuit like the one shown in the following figure.

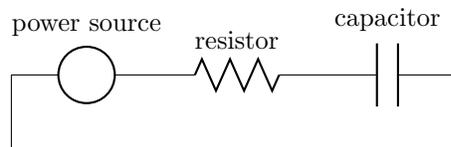


Fig. 3. A resistor/capacitor (RC) circuit.

It has a resistor, a capacitor, and a voltage source: it's an RC circuit. Current flows around the circuit. The current is measured in "amperes" and is denoted by I . (Apparently I stands for the French phrase *intensité de courant*, which was used by Ampère. It is the standard symbol for cur-

rent.) In this "series" circuit, the current is the same everywhere but it may vary with time.

Let's say the positive direction in the circuit is chosen to be clockwise (to the right over the top, for digital clock users). So if current is flowing counterclockwise along the wire, an ammeter would give a negative reading. The system is powered by a variable power source, which creates a "voltage increase" across it. This what makes current move.

3.2. Kirchhoff's Voltage Law

Define $V(t)$ to be the voltage *increase* from the bottom to the top of the source. Define V_R and V_C to be the voltage *drops* across the resistor and capacitor. "Kirchhoff's voltage law" (KVL) states that the total voltage change around a circuit loop is 0, i.e. $V(t) = V_R(t) + V_C(t)$. The graph in the following figure illustrates this.

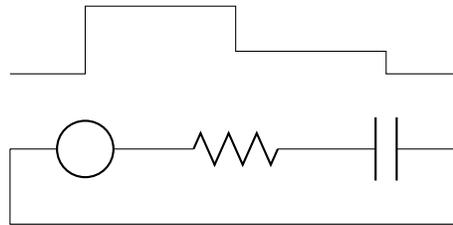


Fig 4. Voltage drops across an RC circuit.

There is a relationship between the voltage drop across each circuit element and the current flowing through it. The relationship is different for resistors and capacitors:

Resistor: $V_R(t) = RI(t)$ for a constant R , the "resistance"

Capacitor: $V_C'(t) = \frac{1}{C}I(t)$ for a constant C , the "capacitance"

(We use V' instead of \dot{V} because the dot does not show up well over the uppercase letter.)

So:

- The voltage drop across the resistor is proportional to the current flowing through it. High resistance means big voltage drop.
- The voltage drop across the capacitor is proportional to the *integral* of the current; it results from a buildup of charge on the two plates of

the capacitor. High capacitance means lots of space for the charge. A very large capacitor C is like no capacitor at all, i.e. $1/C \approx 0$.

To relate these, we differentiate KVL:

$$V'(t) = V_R'(t) + V_C'(t) = RI'(t) + (1/C)I(t)$$

This is a first order linear differential equation for $I(t)$. In standard form:

$$RI'(t) + (1/C)I(t) = V'(t).$$

3.3. Input and Response

For this circuit we will consider the voltage $V(t)$ to be the input signal, the circuit with fixed resistance R and capacitance C to be the system and the current I to be the output signal or system response.

Important note: The term on the right-hand side of the ODE is *not* the actual input signal. Rather it is the *derivative* of the input. We will see many other examples where the input is used to make the right-hand side of the ODE, but is not identical to it.

3.4. Block Diagram

The circuit is the system and it is represented by the left-hand side of the ODE. The input signal is V , the voltage increase across the power source.

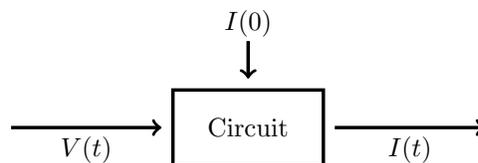


Fig. 5 Block diagram for the RC circuit of Figure 3.

MIT OpenCourseWare
<http://ocw.mit.edu>

18.03SC Differential Equations
Fall 2011

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.