

## Amplitude, Phase, Gain and Bode Plots

We found that the ODE

$$\dot{x} + kx = kB \cos(\omega t) \quad (1)$$

has a particular solution

$$x(t) = \frac{kB}{\sqrt{k^2 + \omega^2}} \cos(\omega t - \phi) = gB \cos(\omega t - \phi) \quad (2)$$

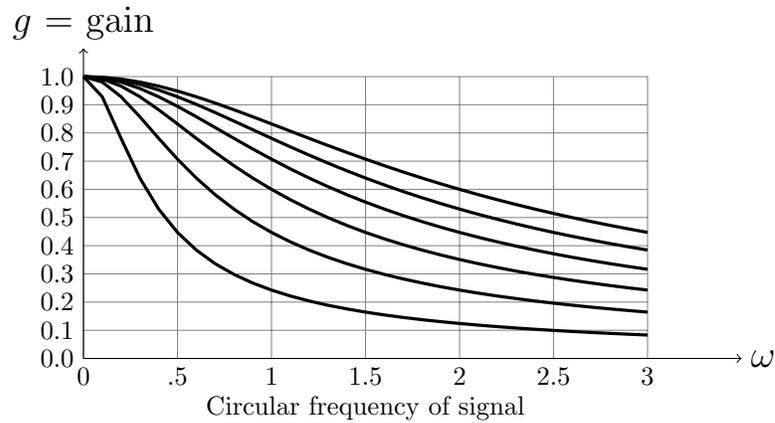
where  $\phi = \tan^{-1}(\omega/k)$ . If we consider the input to be  $B \cos(\omega t)$  then the gain (= output amplitude/input amplitude) is  $g = k/\sqrt{k^2 + \omega^2}$ .

There is a lot more to learn from the formula (2) and its various pieces. The terminology applied below to solutions of the first order equation (1) applies equally well to solutions of second and higher order equations. We will discuss this more when we study second order equations. See also the Mathlet [Amplitude and Phase: First Order](#) for a dynamic illustration.

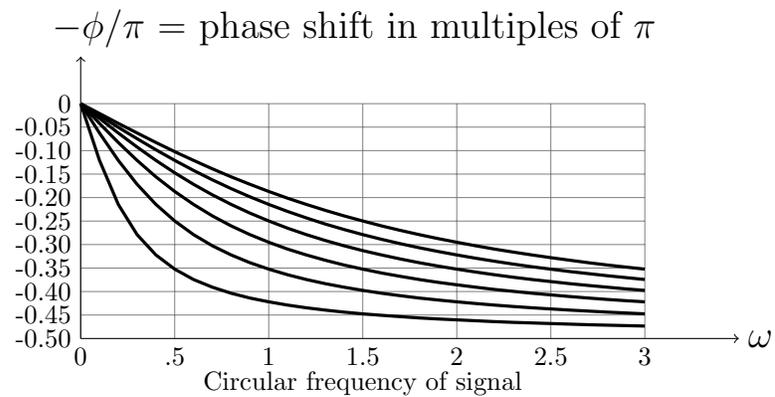
Let's gather all the terminology in one place.

1.  $B \cos(\omega t)$  is the input (or input signal).
2.  $B$  is the input amplitude and  $\omega$  is the input circular frequency.
3.  $x(t)$  is the output or response.
4.  $g = k/\sqrt{k^2 + \omega^2}$  is called the gain or amplitude response. The input amplitude is scaled by the gain to give the output amplitude.
5.  $\phi$  is called the phase lag.

Let's fix the *coupling constant*  $k$  and think about how  $g$  and  $\phi$  vary as we vary  $\omega$ , the circular frequency of the signal. Thus we will regard them as functions of  $\omega$ , and we may write  $g(\omega)$  and  $\phi(\omega)$  in order to emphasize this perspective. We are supposing that we always have the same *system* and are watching its response to a variety of input signals. Graphs of  $g(\omega)$  and  $-\phi(\omega)$  for values of the coupling constant  $k = .25, .5, .75, 1, 1.25, 1.5$  are given below.



**Fig. 1.** First order amplitude response curves



**Fig. 2.** First order phase response curves

These graphs are essentially **Bode plots**. (Technically, the Bode plots display  $\log g(\omega)$  and  $-\phi(\omega)$  against  $\log \omega$ .)

In this course we will focus more on the amplitude response curve (graph of gain vs.  $\omega$ ) than the phase response curve. The phase response is important, we just won't have time to explore it. For equation (1) the amplitude response is rather simple: for any value of  $k$  the gain starts at 1 and decreases to 0 as  $\omega$  goes to infinity.

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