Part I Problems and Solutions

Problem 1: *a) Find a solution of $\dot{x} + 2x = e^{3t}$ of the form Be^{3t}. Then find the general solution.*
b) Now do the same for the complex-valued differential equation $\dot{x} + 2x = e^{3it}$.

Solution: a) Assume $x_p(t) = Be^{3t}$ satisfies $\dot{x} + 2x = e^{3t}$ then substituting this into the DE we get

$$\begin{aligned} & \dot{x} + 2x = e^{3t} \\ \Rightarrow \quad & 3Be^{3t} + 2Be^{3t} = e^{3t} \\ \Rightarrow \quad & 5Be^{3t} = e^{3t} \\ \Rightarrow \quad & 5B = 1 \\ \Rightarrow \quad & B = 1/5. \end{aligned}$$

So, a particular solution is $x_p(t) = \frac{1}{5}e^{3t}$.

The solution to the homogeneous equation $\dot{x} + 2x = 0$ is $x_h(t) = Ce^{-2t}$. The general solution to the original DE is of the form $x = x_p + x_h$, so

$$x = \frac{1}{5}e^{3t} + Ce^{-2t}.$$

b) Similarly, assume $x_p = Be^{3it}$ then substituting this into the DE gives

$$\dot{x} + 2x = B(3i + 2)e^{3it} = e^{3it} \;\Rightarrow\; B = \frac{1}{2 + 3i} = \frac{2 - 3i}{13}.$$

Thus,

$$x_p = \frac{2 - 3i}{13}e^{3it}.$$

The homogeneous solution is the same as in part (a): $x_h = Ce^{-2t}$. Again by superposition the general solution to the DE is

$$x = x_p + x_h = \left(\frac{2 - 3i}{13}\right)e^{3it} + Ce^{-2t}.$$

Remark: This problem is unusual in asking for a complex solution. In this class we will most often ask for the real solution with x_p in amplitude phase form.