

## Complex Arithmetic Examples

In the following we let  $z = 2 + 3i$  and  $w = 4 + 5i$ .

### 1. Real and Imaginary Parts

$$\operatorname{Re}(z) = 2, \quad \operatorname{Im}(z) = 3, \quad \operatorname{Re}(w) = 4, \quad \operatorname{Im}(w) = 5.$$

Note: the imaginary part *does not include*  $i$ .

### 2. Addition and Subtraction

$$\begin{aligned} z + w &= (2 + 3i) + (4 + 5i) = 6 + 8i \\ z - w &= (2 + 3i) - (4 + 5i) = -2 - 2i. \end{aligned}$$

### 3. Multiplication

$$z \cdot w = (2 + 3i)(4 + 5i) = 8 - 15 + i(10 + 12) = -7 + 22i.$$

### 4. Complex Conjugate and Magnitude

$$\begin{aligned} \bar{z} &= \overline{2 + 3i} = 2 - 3i \\ |z| &= \sqrt{4 + 9} = \sqrt{13} \\ z + \bar{z} &= 2 + 3i + 2 - 3i = 4 = 2 \operatorname{Re}(z) \\ z \cdot \bar{z} &= (2 + 3i)(2 - 3i) = 4 + 9 = 13 = |z|^2 \end{aligned}$$

### 5. Division

Multiply numerator and denominator by the complex conjugate of the denominator:

$$\frac{w}{z} = \frac{4 + 5i}{2 + 3i} = \frac{4 + 5i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{8 + 15 + i(-12 + 10)}{13} = \frac{23}{13} - \frac{2}{13}i.$$

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18.03SC Differential Equations  
Fall 2011

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