18.03SC Differential Equations, Fall 2011
Transcript – Amplitude and Phase: First Order Applet

PROFESSOR: I want to use this applet, Amplitude And Phase, First Order, to show you some of the functionalities that are common to the MIT mathlets.

First of all, if your browser window looks like this, so that the label at the top is too large, or like this, so that it's missing some of the window, it's because you have the zoom on your browser set wrong. And you can adjust it with command, or control plus or minus keys, so that it looks correct.

So let's see what we have in this applet. At the top, you can see the differential equation that this applet deals with. This differential equation models many different physical phenomena, and the story I want to tell you involves the ocean, measured by this blue level here, connected to a bay by this channel. And here's a measure of the water level in the bay.

As time goes on, the level of water in the ocean changes because of tides, and the water level in the bay follows suit mediated by a slanting level in the channel. I'm dragging this time slider under the graphing window, and you can set it wherever you want. In fact, if you want to set it at exactly 12, because there's a hash mark here, I can click on this 12, and the time slider moves to that point.

Or you can return time to time zero by this arrow key, and then animate by pushing this double right arrow key and watch the tide move up and down.

There's very carefully designed color coding in all of these applets. In this case, blue represents the ocean, and you can see that it also represents the curve traced out by the height of the ocean as time increases. And yellow represents the water level in the bay.

When I drag the cursor over this graphing window, you can see a crosshair forming, and below the window, you can see readout of time and the x variable, vertical direction.

You can use this to make measurements. For example, I can see that the tide in the bay seems to have maximal height of 0.38. In many of these applets, you can make measurements by using a rollover over the graphing window.

Let's see what else we have in this applet. Down here, there's a slider marked k. This is the coupling constant. It reflects the width of the channel in the story that I'm telling you. When k is small, the channel is very narrow and the ocean level has very little effect on the water level in the bay.

In fact, when k is equal to zero, it has no effect at all. On the other hand, when k becomes larger, the water level in the bay tracks the ocean water level very closely. Let's set k to one, here.

And think about another thing you can vary. In reality, you can't change the period of the tides. Even King Canute couldn't do that. But here in this tool, we can. And I can vary the circular frequency of this tidal input signal by changing the omega slider over here on the right. I can make it small, and so the period is very long, or larger and make the tides happen faster.

So we can animate this again. You can see things happen faster. The curve is tighter, and the effect is different. In fact, if we watch what happens when I move omega from one down to a smaller number, you can see the maximum height of the tide changes. The amplitude of the tide changes. It depends upon omega.

In fact, one of the nice things about these systems is that if you have a sinusoidal input-- as we do here, this blue curve-- then the output signal is also going to be sinusoidal and of the same frequency. So there's only two things we need to know about the output here, this yellow signal. Namely, the amplitude and the phase lag behind the input signal, the blue curve.

And those two quantities can be measured, can be graphed, against the input frequency, and we can see those graphs by clicking on this check box called Bode plots down here.

So this opens two new windows. The upper one records the amplitude as a function of the angular input frequency, and the lower one records the number of degrees behind the input frequency that the bay falls. So as omega changes, you can see all these various things changing simultaneously.

This is a characteristic feature of these applets. The same information is recorded in several different places and always connected visually by placement. So you can see the amplitude is 0.71 here. This yellow horizontal line connects it with the maxima of the output curve. And also by color.

What else do we see here? There's a red line segment here on this curve, which I can make a little bit bigger if I decrease the k a bit and decrease the frequency a little bit.

What is this red curve here, this red line here? Well, it connects to this vertical strut which goes up to the maximum of the output curve. And it begins at the maximum of the input curve. In other words, it's the amount of time that the output falls behind the input. It's the time lag, and that time lag is recorded numerically down here. It's 1.96 in this example.

There's one more check box to explore called Nyquist plot here. Let's click it and see what happens.

This opens a window at the bottom here which records a complex number. That complex number contains a magnitude and an angle. And the magnitude is the amplitude of the system response, and the angle represents the phase lag of the system response.

You can see these things changing together when I move the omega slider here. Watch both the windows above the slider and the window below the slider, and you can see that they change together.

This bottom representation is a complex number. It's very useful in understanding the relationship between phase lag and amplitude, and it also represents the way we solve these differential equations in the Differential Equations course at MIT.

This is just a beginning and indication of some of the functionalities of these MIT mathlets. If you want to see a list of them that are associated any one of the applets, there's always a Help key in the upper right hand corner which will open a page that simply describes the functionalities present in that particular applet.

So have fun with these. You can play around with them. You can't break anything by clicking buttons and experimenting with rollovers and moving the sliders.