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18.034 Honors Differential Equations
Spring 2009

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18.034 Recitation: April 14th, 2009

1. Consider the problem of recovering the differential operator $T = p(D)$ from knowledge of the solution $y(t)$ to $Ty = 0$ with initial conditions

$$y(0) = 1, \quad y'(0) = y''(0) = \dots = y^{(n-1)}(0) = 0.$$

(Since $p(D)$ can at most be determined up to a multiplicative constant, we assume $p(D)$ to be monic.)

- (a) Check that $\mathcal{L}[y^{(j)}] = s^j \mathcal{L}[y] - s^{(j-1)}$ for $j = 1, 2, \dots, n$, and conclude that $Y(s)$ gives no information about $P(s)$ if $P(0) = 0$ (except that, in fact, $P(0) = 0$).
 - (b) Show that if $P(0) \neq 0$, however, $P(s)$ can be fully determined from $Y(s)$.
2. Suppose $A : (a, b) \rightarrow M_n$ is an $n \times n$ matrix and $\det A(t) \neq 0$ for all $t \in I$. Compute $B'(t)$ where $B(t) = A^{-1}(t)$.
 3. Take the second order equation $x'' + p(t)x' + q(t)x = 0$ and change it to a first-order system $\mathbf{v}' = A\mathbf{v}$. in the usual way. Show that the Wronskian of two solutions x_1, x_2 to the original equation is the same as the Wronskian of the two corresponding solutions $\mathbf{x}_1, \mathbf{x}_2$ of the system. Show that Abel's formula for the Wronskian of two (scalar) solutions to the second order ODE agrees with what one obtains for the Wronskian of two linearly independent solutions in the vector formulation when one applies the identity $(\det Y(t))' = \det Y(t) \operatorname{tr} Y'$ for an smooth invertible family of matrices $Y(t)$.

4. For the system

$$y_1' = 3y_1 + 2y_2, \quad y_2' = -2y_1 - y_2,$$

find the unique fundamental matrix $U(t)$ satisfying $U(0) = I$.