18.034 Honors Differential Equations
Spring 2009

18.034 Recitation: April 9th, 2009

1. Consider the differential equation $y'' + y = h(t) - h(t-c)$ for $c > 0$.

    (a) Use the Laplace transform to find the rest solution.

    (b) Show that y and y' are continuous at $t = c$ but y'' is not.

2. Consider the equation $y^{(n)} = \delta$, where $y(t) = 0$ for $t < 0$. Suppose that y is "maximally regular" at 0, i.e., as many derivatives of y as possible are continuous at 0. Show that $y^{(n-1)}$ has a jump of magnitude 1 at $t = 0$.

3. (a) For $n \geq 0$, what is the action of the distribution $\delta^{(n)}$ on a test function ϕ?

    (b) Explore the continuity of rest solutions to $y'''(t) = f(t)$ for the choices t, 1, $\delta(t)$, $\delta'(t)$, δ'' of $f(t)$.

4. The following *boundary-value problem* models the equation of the central line $y(x)$, $0 \leq x \leq 2$, of a uniform weightless beam anchored at one end and carrying a concentrated load at its center.

$$y^{(iv)} = 6\delta(x-1), \quad y(0) = y'(0) = y''(2) = y'''(2).$$

    (a) With $y''(0) = 2a$, $y'''(0) = 6b$, find y via $Y(s)$.

    (b) Determine a and b from the boundary conditions at $x = 2$.