18.034 Honors Differential Equations
Spring 2009

18.034 Recitation: March 12th, 2009

1. Find the general solution to the following equations via an explicit variation of parameters procedure.

   (a) $y'' + y = 1$

   (b) $xy'' - y' = 1$.

   (Hint: In the first case, consider $y = a(x)\sin x + b(x)\cos x = 1$*, in the second,* $y = a(x) + b(x)x^2$*.)*

2. (Birkhoff-Rota, p. 62, #3)

   Solve
   $$y'' + 3y' + 2y = x^3$$
   for the initial conditions $y'(0) = y(0) = 0$.

3. (Birkhoff-Rota, p. 62, #5d)

   Construct a Green's function for the initial-value problem associated to the ODE
   $$x^2u'' - (x^2 + 2x)u' + (x + 2)u = 0.$$
   Hint: $u(x) = x$ *is a solution.*

4. (Birkhoff-Rota, p. 63, #8) Show that, if $q(t) < 0$, the Green's function $G(t, \tau)$ for the initial-value problem associated to $u'' + q(t)u = 0$ is positive and convex upward for $t > \tau$.

5. (Birkhoff-Rota, #5, p. 82)

Show that every linear differential operator L with constant *real* coefficients can be factored as $L = AL_1 \circ L_2 \circ \cdots \circ L_m$ where $A \in \mathbb{R}$ and $L_i = D_i + b_i$ or $L_i = D^2 + p_i D + q_i$.