18.034 Honors Differential Equations
Spring 2009

18.034 Recitation: March 10th, 2009

1. Let $I \subset \mathbb{R}$ be an interval and recall that a function $f : I \to \mathbb{R}$ is said to be *Lipschitz* on I if there is a constant C such that $|f(x) - f(y)| \leq C|x - y|$ for all x, $y \in I$.

   (a) Show that if f is Lipschitz on I, then $f \in C(I)$.

   (b) Show that if f is differentiable on I with bounded derivative, then f is Lipschitz on I.

   (c) Show that $f(x) = |x|$ is Lipschitz on $\mathbb{R}$.

   (d) Show that $f(x) = e^x$ is not Lipschitz on $\mathbb{R}$.

2. (Birkhoff-Rota, p. 62, #3)

   Solve

   $$y'' + 3y' + 2y = x^3$$

   for the initial conditions $y'(0) = y(0) = 0$.

3. (Birkhoff-Rota, p. 62, #4)

   Show that any second-order linear inhomogeneous equation that has both x^2 and $\sin^2 x$ as solutions must have a singular point at the origin.

4. Describe the dominant behavior as $t \to \infty$ of any solution u to

   $$(D + 3)(D^2 + 1)^5 u = 640 \cos t.$$

   Hint: the function

   $$U_0(t) = \frac{t^5}{5!}(6 \sin t - 2 \cos t)$$

   solves the equation.