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18.034 Honors Differential Equations  
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### 18.034 Practice Midterm #3

Notation.  $' = d/dt$ .

1. (a) If  $f \in E$  and  $F(s) = \mathcal{L}[f(t)]$ , show that  $\lim_{s \rightarrow \infty} F(s) = 0$ .

(b) Find the inverse Laplace transform of  $F(s) = \log \left( \frac{s+1}{s-1} \right)$ .

2. (a) Sketch the graph of  $f(t) = (1/5)(h(t-5)(t-5) - h(t-10)(t-10))$ , where  $h(t)$  is the unit step function or the Heaviside function.

(b) Find the solution of the initial value problem

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = 0.$$

Sketch the graph of the solution.

(c) Compute the left and the right limits of  $y''(t)$  at  $t = 5$  and  $t = 10$ .

3. Consider two vectors  $\vec{y}_1(t) = (t, 1)$  and  $\vec{y}_2(t) = (t^2, 2t)$ .

(a) In which intervals are  $\vec{y}_1$  and  $\vec{y}_2$  linearly independent?

(b) Find a system of differential equations satisfied by  $\vec{y}_1$  and  $\vec{y}_2$ .

4. Find the general solution of

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

5. Let

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

(a) Show that  $A^2 = -I$ .

(b) Show that

$$e^{At} = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}.$$

(c) Find the general solution of

$$\begin{pmatrix} x \\ y \end{pmatrix}' = A \begin{pmatrix} x \\ y \end{pmatrix}.$$

(d) Sketch solutions in the  $(x, y)$ -plane and discuss their behavior.