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18.034 Honors Differential Equations  
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## 18.034 Practice Midterm #1

1. Solve the initial value problem

$$y'' - 3y^2 = 0, \quad y(0) = 2, y'(0) = 4.$$

Determine the interval in which the solution exists.

2. Consider the differential equation  $y' = (1 - y)(y - 2)^3$ .

(a) Sketch the graph of  $f(y) = (1 - y)(y - 2)^3$ .

(b) Determine the critical points (stationary solutions).

(c) Discuss the stability of critical points in part (b).

3. Determine the values of  $a$ , if any, for which all solutions of the differential equation

$$y'' - (2a - 1)y' + a(a - 1)y = 0$$

tend to zero as  $t \rightarrow \infty$ . Also, determine the values of  $a$ , if any, for which all (nonzero) solutions become unbounded as  $t \rightarrow \infty$ .

4. Consider the undamped forced vibration system

$$y'' + \omega^2 y = \sin 2t, \quad u(0) = 0, \quad u'(0) = 0.$$

(a) Find the solution for  $\omega \neq 2$ .

(b) Find the solution for  $\omega = 2$ .

5. (a) Find the value  $m$  for which  $y = t^m$  is a solution of the differential equation

$$t^2 y'' - 13ty' + 49y = 0, \quad t > 0.$$

(b) Find a second solution of the differential equation in part (a).

6. Show that every solution of  $u'' + (1 + e^t)u = 0$  vanishes infinitely often on  $0 < t < \infty$ .