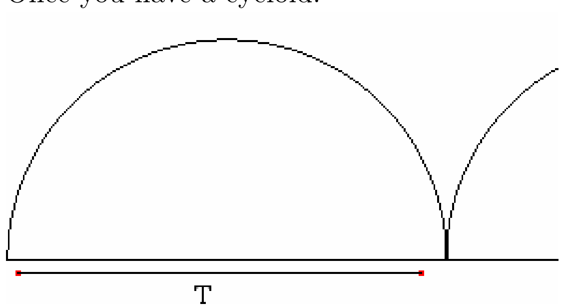
18.034 Honors Differential Equations
Spring 2009

Problem set 7, Solution keys

1. (a) -
   (b) $Y(s) = \frac{P_1(s)}{P_2(s)} F(s) = W(s)F(s)$. $y = w * f$
   (c) Because $W(s) \to 0 \quad \text{as} \quad s \to \infty$.
   If $P_1 = P_2$ then $W(s) = 1$ and $w(t) = \delta(t)$. So, $y = f$.

2. (a) -
   (b) Once you have a cycloid:



   $T = 2\pi \frac{E}{WH}$ is calculated by mesuring the distance between two cusps.

3. (a) $(UC)' = U'C = (AU)C = A(UC)$ and $|UC| = |U||C| \neq 0$
   (b) $V(t_0)$ is a non-singular matrix.
   (c) Let $Y = (\vec{y_1}\vec{y_2})$.

$$\begin{aligned} |Y|' &= |\vec{y_1}'\vec{y_2}| + |\vec{y_1}\vec{y_2}'| \\ &= |A\vec{y_1}\vec{y_2}| + |\vec{y_1}A\vec{y_2}| \\ &= D_1 + D_2 = (a_{11} + a_{22})|Y|. \qquad \text{Liouviue's theorem.} \end{aligned}$$

4. (a) -
   (b) By setting $\phi(t) = t^m$, $\begin{pmatrix} 2 & t^2 \\ t & t^3 \end{pmatrix}$.
   (c) By setting $\phi'(t) = e^{\frac{2}{t}}$, $\begin{pmatrix} 1 & \phi(t) \\ t & t\phi(t) - \frac{t^2}{2}e^{\frac{2}{t}} \end{pmatrix}$

5. (a) $c_1 e^{3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
   (b) $\begin{pmatrix} a \\ b \end{pmatrix} = c \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, c is an arbitrary const.

6. $\begin{pmatrix} a \\ 0 \\ c \end{pmatrix}$, a and c are arbitrary.