18.034 Honors Differential Equations
Spring 2009

18.034 Solutions to Problemset 2

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1. $y(x) = \begin{cases} -x + c_1 & x < 0 \\ x + c_2 & x > 0 \end{cases}$ for some c_1 and c_2.

   y is continuous at $x = 0 \Rightarrow c_1 = c_2 = 0$.

   But $y(x) = \begin{cases} -x + c_1 & x < 0 \\ x + c_2 & x > 0 \end{cases}$ is <u>not</u> differentiable at $x = 0$.

2. As long as the solution is defined, $\frac{y'}{y} = f(y)$ (y is never zero by uniqueness).

   $\Rightarrow (\log |y|)' \leq |f(y)| \leq M$ by assumption. Then

   $$|y(x)| \leq |y_0| e^{Mx} \tag{1}$$

   For $a > 0$, consider the rectangle $\{(x, y) : |x| \leq a, |y| \leq |y_0| e^{Ma}\}$. If the solution does <u>not</u> exist on $x \in (-a, a)$, then $|y(a)| = |y_0| e^{Ma}$. This contradicts (**??**).

3. Let $\alpha = a + ib$, $\beta = c + id$. In terms of polar coordinate functions,

   $$y'' = \mathrm{Im}(\alpha\beta) \frac{\cos(\theta)\,\mathrm{Re}((\beta + i\alpha)e^{i\theta})}{\mathrm{Re}(\alpha e^{-i\theta})}$$

   So y'' changes signs at slopes $-b/a$, ∞ and $\frac{b-c}{a+d}$.

4. (a) u solves the DE $y' + (-b(x) + 2c(x)y_1(x))y + c(x)y^2 = 0$.

   (b) $y_1(x) = x$, $u(x) = -\frac{1}{x+c}$.

5. (a) $c_1 \sin x + c_2 \cos x$

   (b) $-\sin 2x$, 3, $2e^x$

   (c) $-b \sin x - \sin 2x + 3 + 2e^x$

6. (a) $\ddot{u} + (p-1)\dot{u} + qu = 0$, $\cdot = \frac{d}{dt}$

   (b) $\sin\log|x|$, $\cos\log|x|$

   (c) No solutions