18.034 Honors Differential Equations
Spring 2009

18.034 Problem Set #3

Due by Friday, March 6, 2009, by NOON.

1. This problem pertains to the differential equation $y'' + \omega^2 y = \sin\omega_0 t$, where $\omega \neq 0$ and ω_0 is close to but different from ω.

(a) Verify that $y_1(t) = \dfrac{\sin\omega_0 t}{\omega^2 - \omega_0^2}$ is a particular solution.

(b) As $\omega_0 \to \omega$ show that one of the initial conditions $y_1(0)$ or $y_1'(0)$ becomes infinite.

(c) Check that $y_2(t) = \dfrac{\sin\omega_0 t - \sin\omega t}{\omega^2 - \omega_0^2}$ is the particular solution for which the initial conditions remain finite as $\omega_0 \to \omega$.

(d) By l'Hospital's rule show that the limit as $\omega_0 \to \omega$ of $y_2(t)$ gives a particular solution of $y'' + \omega^2 y = \sin\omega t$.

2. Let $f(x)$ and $g(x)$ be two solutions of the differential equation $y' = F(x, y)$ in a domain where F satisfies the condition*:

$$y_1 < y_2 \quad \text{implies} \quad F(x, y_2) - F(x, y_1) \le L(y_2 - y_1).$$

Show that

$$|f(x) - g(x)| \le e^{L(x-a)}|f(a) - g(a)| \qquad \text{if} \quad x > a.$$

3. Very that $(\sin x)/x$, x satisfy the following equations, respectively, and thus obtain the second solution.

(a) $xy'' + 2y' + xy = 0 \qquad (x > 0)$,

(b) $(2x - 1)y'' - 4xy' + 4y = 0 \qquad (2x > 1)$.

4. (a) Birkhoff-Rota, pp. 57, #4. (Typo. $I(x) = q - p^2/4 - p'/2$.)

(b) Birkhoff-Rota, pp. 57, #7(a). (Use part (a) instead of #6 as is suggested in the text.)

(c) Birkhoff-Rota, pp. 57, #7(b).

5. Let $(\cosh x)y'' + (\cos x)y' = (1 + x^2)y$ for $a < x < b$ and let $y(a) = y(b) = 1$. Show that $0 < y(x) < 1$ for $a < x < b$.

6. (a) Birkhoff-Rota, pp. 75, #3, (b) Birkhoff-Rota, pp. 75, #4.

*It is called a *one-sided Lipschitz condition*.