18.034, Honors Differential Equations
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Lecture 35
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1. Began by defining (again) equilibrium solution, nondegenerate equilibrium and degenerate equilibrium. Then I defined stable, asymptotically stable, neutrally stable, unstable, attractor, repeller and basin of attraction. If for every initial value x_0, the solution curve x(t) is defined for all $t > 0$, defined the maps $\Phi_t : R \to R$ by $\Phi_t(x_0) = x(t)$. Proved $\Phi_{t_1}(\Phi_{t_0}(x)) = \Phi_{t_1+t_0}(x)$ (so this is a flow). Pointed out that for a linear system, $\Phi_t = \exp(tA)$.

2. Proved basic thms about equilibrium points of a linear system.

   Rmk: The equilibrium points of $x' = Ax$ are precisely nullvectors of A. Up to a translation, each nullvector is equivalent to the origin.

   Thm 8.1.3: The system $x' = Ax$ is stable at the origin iff
   (1) every eigenvalue has nonpositive real part
   (2) for each eigenvalue $\lambda = i\beta$ (possibly $\beta = 0$), the eigenspace is not deficient.

   (B) The system is asymptotically stable at the origin iff every eigenvalue has negative real part.

3. For a 2x2 linear system, went through most of the case (depending on whether det(A) is >0, =0, <0, trace(A) is >0, =0, <0 and whether 4det(A)-trace(A)^2 is >0, =0, <0).