18.034, Honors Differential Equations
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Lectures 33 + 34 4/28 + 4/30/04

1. Began to discuss nonlinear systems and the relation to linear systems, e.g. for a pendulum, $\ddot{\Theta} = \omega^2 \sin(\theta)$, but for Θ and t small, approximately the same as $\ddot{\Theta} = \omega^2 \theta$.

2. Defined "structurally stable" (I'm not certain what the conventional term is). Given a nonlinear system $x' = F(x,t)$ (or just = $F(x)$ for an autonomous system), a property of the system is structurally stable if for every continuous $G(x,t)$ (or just G(x) for an autonomous system), there exists $\varepsilon_0 = \varepsilon_0(F,G)$ such that the property holds for $x' = F + \varepsilon G$, $|\varepsilon| < \varepsilon_0$.

Gave the example of the number of equilibrium points.

Prop: Let R C $\mathbb{R}^n$ be a bounded closed region. If

(1) these are no equilibrium points or ∂R

(2) these are only finitely many equilibrium points in Int (R),

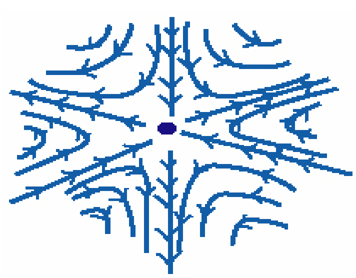
(3) every equilibrium point is nondegenerate, i.e. $\left|\frac{\partial F_i}{\partial R_j}\right| \neq \varepsilon$ at the equilibrium point, then $\exists \varepsilon_0 > 0$ such that for all $-\varepsilon_0 < \varepsilon < \varepsilon_0$, (1), (2) +(3) hold for $F + \varepsilon G$. Moreover the number of equilibrium points is constant.

Pf :- Use the implicit function theorem for $F + \varepsilon G = R \times \mathbb{R} \rightarrow W$

3. Defined orbits + orbital portrait for a system (obviously closely related to the orbital portraits from Ch.3)

Defined nullclines. Worked through the example $\begin{cases} x' = xy \\ y' = x^2 - y^2 \end{cases}$

Guess these are solutions $y = mx$ and solve to get $m^2 = \frac{1}{2}$



Orbits look roughly like the contour curves of a monkey saddle.

4. Gave algorithm for sketching an orbital portrait for an autonomous 2D system.

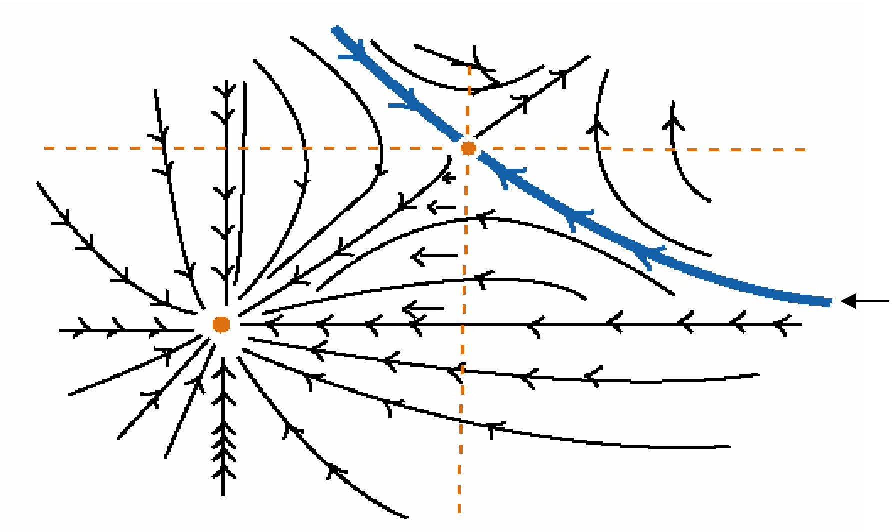
Step 1 : Find all equilibrium points.

Step 2 : For each equilibrium point, draw the "local picture" (if it is nondegenerate + structurally stable)

Step 3 : Draw the nullclines and other "fences" (i.e. curves that help to determine basins of attraction).

Step 4 : Interpolate between the local pictures to give a rough sketch.

Went through the steps for $\begin{cases} x' = x(y-1) \\ y' = y(x-1) \end{cases}$



Separatrix separating basins of attraction.