18.034, Honors Differential Equations
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Lecture 25
4/7/04

1. Discussed the approach to the Green's Kernel via the Laplace operator. Let p(D) be a constant coeff. linear differential operator of order n+1 and let f(t) be a function of exponential type. Let y(t), $t \geq 0$ be the solution of the IVP

$$\begin{cases} p(D)y = f(t) \\ y(0) = y_0 \\ \vdots \\ y^{(n)}(0) = y_0^{(n)} \end{cases}$$ Let $y_n(t)$ be the sol'n of $$\begin{cases} p(D)y = 0 \\ y(0) = y_0 \\ \vdots \\ y^{(n)}(0) = y_0^{(n)} \end{cases}$$

let k(t) be the sol'n of $$\begin{cases} p(D)y = 0 \\ y(0) = 0 \\ y^{(n-1)}(0) = 0 \\ y^{n}(0) = 1 \end{cases}$$, and let $y_p(t)$ be the sol'n of

$$\begin{cases} p(D)y = f(t) \\ y(0) = 0 \\ y_0^{(n)}(0) = 0 \end{cases}$$ Then we have $p(s)\ L[y] - Q(s) = F(s)$ for some poly. Q(s) of degree $\leq$ n and

$F(s) = L[f(t)]$. So $L[y] = \dfrac{Q(s)}{p(s)} + \dfrac{1}{p(s)}F(s)$. Moreover, we also have

$L[y_n] = \dfrac{Q(s)}{p(s)}$ and $L[k] = \dfrac{1}{p(s)}$. Therefore,

$y(t) = y_n(t) + \int_0^t k(t-u)f(u)du$. Moreover $y_p(t) = \int_0^t k(t-u)f(u)du$

2. Worked the IVP $$\begin{cases} y'' + 2y' + y = f(t) = te^{2t} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$ by Green's Kernel method

   And saw it involved <u>more</u> computation than the usual Laplace operator method (which we did in lecture on Monday).

3. Very quickly reviewed what a system of linear ODE's is , introduced matrix notation for such a system,

$$y' = Ay + F(t)$$

   and argued that a formal solution should be of the form

$$y = \exp[tA] \cdot y_0 + \exp[tA] \cdot \int_0^t \exp[-uA]F(u)du$$, once we make sense of all this.