18.034, Honors Differential Equations
Prof. Jason Starr
Lecture 17
3/12/04

1. Philosophy about what Fourier series are supposed to do and why this is useful for us:

   $$y'' + ay + by = f(t)$$

   If f(t) is periodic of period 2L, write

   $$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi t}{L}\right) + B_n \sin\left(\frac{n\pi t}{L}\right).$$

   Solve each driven ODE $y_n'' + ay_n' + by_n = A_n \cos\left(\frac{n\pi t}{L}\right)$, etc. and use superposition to find sol'n of original ODE. Useful for analysis of resonant frequency, etc.

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2. Defined spaces of real/complex-valued k-times continuously differentiable/ piecewise k-times cts. diff. functions $C_{\mathbb{R}}^t[a,b]$, $C_{\mathbb{C}}^t[a,b]$, $PC_{\mathbb{R}}^t[a,b]$, $PC_{\mathbb{C}}^t[a,b]$.

3. Defined the inner product $< f,g > = \int_a^b f(t)\overline{g}(t)dt$ and talked about properties (= axioms for Hermitian inner product space). Defined $\|f\| = \sqrt{< f, f >}$, $\partial_{mean}(f,g) = \|f-g\|$. Mention this is different than the uniform metric.

4. Defined orthogonal and orthonormal sequences.
   Checked that $\{1\} \cup \left\{\cos\left(\frac{n\pi t}{L}\right), \sin\left(\frac{n\pi t}{L}\right)\right\}_{n=1,2,....}$ gives an orthogonal sequence and computed the norms.

5. Posited existence of a Fourier series
   $f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi t}{L}\right) + B_n \sin\left(\frac{n\pi t}{L}\right)$ for f(t)+$PC_{\mathbb{R}}$[-L,L]
   Concluded $A_0 = \frac{1}{2L}\int_{-L}^{L} f(t)dt$, $A_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi t}{L}\right)dt$, $B_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin dt$