18.034 EXAM 3
APRIL 26, 2004

Name: ______________________________________

Problem 1:	__________	/30
Problem 2:	__________	/20
Problem 3:	__________	/25
Problem 4:	__________	/25
Total:	__________	/100

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, closed notes, and calculators are not allowed. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Date: Spring 2004.

Table of Laplace Transforms

	$y(t)$	$Y(s) = \mathcal{L}[y(t)]$
1.	$y^{(n)}(t)$	$s^n Y(s) - (y^{(n-1)}(0) + \cdots + s^{n-1} y(0))$
2.	t^n	$n!/s^{n+1}$
3.	$t^n y(t)$	$(-1)^n Y^{(n)}(s)$
4.	$\cos(\omega t)$	$s/(s^2 + \omega^2)$
5.	$\sin(\omega t)$	$\omega/(s^2 + \omega^2)$
6.	$e^{at} y(t)$	$Y(s-a)$
7.	$y(at),\ a > 0$	$\frac{1}{a} Y(s/a)$
8.	$S(t-t_0) y(t-t_0),\ t_0 \geq 0$	$e^{-st_0} Y(s)$
9.	$\delta(t-t_0),\ t_0 \geq 0$	e^{-st_0}
10.	$(S(t)y) * (S(t)z)$	$Y(s)Z(s)$
11.	$y(t),\ y(t+T) = y(t)$	$\frac{1}{1-e^{-sT}} \int_0^T e^{-st} y(t) dt$

Name: ______________________________ **Problem 1**: ____________ /30

Problem 1(30 points) Consider the following inhomogeneous 2^{nd} order linear differential equation,

$$\begin{cases} y'' - 3y' + 2y = e^{2t}, \\ y(0) = 0, \\ y'(0) = 0 \end{cases}$$

Denote by $Y(s)$ the Laplace transform,

$$Y(s) = \mathcal{L}[y(t)] = \int_0^\infty e^{-st} y(t) dt.$$

(a)(5 points) Taking the Laplace transform of both sides of the ODE, find an equation that $Y(s)$ satisfies.

(b)(15 points) Determine the partial fraction expansion of $Y(s)$.

(c)(10 points) Determine $y(t)$ by computing the inverse Laplace transform of $Y(s)$.

Name: ______________________________________ **Problem 2**: ________________ /20

Problem 2(20 points) A real $n \times n$ matrix is *symmetric* if it is equal to its transpose, i.e. $A = A^T$. The general real symmetric 2×2 matrix is,

$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}.$$

(a)(10 points) Let A be the real symmetric 2×2 matrix above. Compute the trace, determinant and characteristic polynomial of A.

(b)(10 points) Prove that every eigenvalue of the real symmetric 2×2 matrix A is real.

Extra credit(15 points) Let A be a real symmetric $n \times n$ matrix, where n is arbitrary. Prove that every eigenvalue of A is real.

Name: ______________________________________ **Problem 3**: ________________ /25

Problem 3(25 points) For each of the following matrices A, compute the following,

(i) Trace(A),
(ii) $\det(A)$,
(iii) the characteristic polynomial $p_A(\lambda) = \det(\lambda I - A)$,
(iv) the eigenvalues of A (both real and complex), and
(v) for each eigenvalue λ a basis for the space of λ-eigenvectors.

(a)(15 points) The 2×2 matrix with real entries,

$$A = \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix}.$$

(b)(10 points) The 3×3 matrix with real entries,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Name: ______________________________ **Problem 4**: ____________ /25

Problem 4(25 points)

Let $f(t)$ be a function of exponential type, let $\mathbf{f}(t)$ be the following column vector,

$$\mathbf{f}(t) = \begin{bmatrix} 0 \\ f(t) \end{bmatrix},$$

and let A be the following 2×2 matrix,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Consider the following inhomogeneous system of 1$^{\text{st}}$ order linear ODEs,

$$\begin{cases} \mathbf{y}'(t) = A\mathbf{y}(t) + \mathbf{f}(t), \\ \mathbf{y}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{cases}$$

Denote by $\mathbf{Y}(s)$ the Laplace transform of $\mathbf{y}(t)$, i.e.,

$$\mathbf{Y}(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}, \quad Y_i(s) = \mathcal{L}[y_i(t)], \ i = 1, 2.$$

Denote by $F(s)$ the Laplace transform of $f(t)$.

(a)(5 points) Express $\mathcal{L}[\mathbf{y}'(t)]$, $\mathcal{L}[A\mathbf{y}(t)]$ and $\mathcal{L}[\mathbf{f}(t)]$ in terms of $\mathbf{Y}(s)$ and $F(s)$.

(b)(10 points) Using part (a), find an equation that $\mathbf{Y}(s)$ satisfies, and iteratively solve the equation for $Y_2(s)$ and $Y_1(s)$, in that order.

(c)(10 points) Determine $y_1(t)$ and $y_2(t)$ by applying the inverse Laplace transform to $Y_1(s)$ and $Y_2(s)$. Your answer should be expressed in terms of a convolution involving $f(t)$.