Recitation 11, March 11, 2010

Frequency Response

This project will be much more meaningful if it is accompanied by the Mathlet `Amplitude and Phase: Second order, IV` (available at http://math.mit.edu/daimp/). This illustrates the second order mass/spring/dashpot system driven by a force F_{ext} acting directly on the mass: $m\ddot{x} + b\dot{x} + kx = F_{\text{ext}}$. So the input signal is F_{ext} and the system response is x. We're interested in sinusoidal input signal, $F_{\text{ext}}(t) = A\cos(\omega t)$, and in the steady state, sinusoidal system response, $x_p(t) = gA\cos(\omega t - \phi)$. Here g is the gain of the system and ϕ is the phase lag. Both depend upon ω, and we will consider how. We might as well take $A = 1$, so the amplitude of the system response equals the gain.

Take $m = 1$, $b = \frac{1}{4}$, and $k = 2$.

1. Compute the complex gain $H(\omega)$ of this system. (This means: make the complex replacement $F_{\text{cx}} = e^{i\omega t}$, and express the exponential system response z_p as a complex multiple of F_{cx}: $z_p = H(\omega)F_{\text{cx}}$.)

2. Write down the expression for the gain $g(\omega) = |H(\omega)|$. What is the amplitude of the system response when $\omega = 1$? (You can check your answer using the applet.)

3. What is the resonant circular frequency ω_r? (Hint: minimize the square of the denominator.)

4. It appears that the phase lag is approximately $\frac{\pi}{2}$ at the resonant circular frequency. Is that correct? That is, at what frequency is the phase lag equal to one quarter cycle?

5. At what circular frequency is the phase lag equal to $\frac{\pi}{4}$? How about $\frac{3\pi}{4}$?

6. New project: Find a solution of $\ddot{x} + 3\dot{x} + 2x = te^{-t}$.

18.03 Differential Equations
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