Problems: Jacobian for Spherical Coordinates

Use the Jacobian to show that the volume element in spherical coordinates is the one we've been using.

Answer: $z = \rho\cos\phi, \quad x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta$

$$\begin{aligned}
\Rightarrow \quad \frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} &= \begin{vmatrix} \sin\phi\cos\theta & \rho\cos\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \sin\phi\sin\theta & \rho\cos\phi\sin\theta & \rho\sin\phi\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{vmatrix} \\
&= \cos\phi \begin{vmatrix} \rho\cos\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \rho\cos\phi\sin\theta & \rho\sin\phi\cos\theta \end{vmatrix} + \rho\sin\phi \begin{vmatrix} \sin\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \sin\phi\sin\theta & \rho\sin\phi\cos\theta \end{vmatrix} \\
&= \cos\phi(\rho^2\sin\phi\cos\phi) + \rho\sin\phi\,\rho\sin^2\phi \\
&= \rho^2\sin\phi(\cos^2\phi + \sin^2\phi) \\
&= \rho^2\sin\phi \qquad \text{(as promised.)}
\end{aligned}$$