

Problems: Green's Theorem and Area

1. Find M and N such that $\oint_C M dx + N dy$ equals the polar moment of inertia of a uniform density region in the plane with boundary C .

Answer: Let R be the region enclosed by C and ρ be the density of R . The polar moment of inertia is calculated by integrating the product mass times distance to the origin:

$$I = \iint_R dI = \iint_R r^2 dm = \iint_R (x^2 + y^2) \cdot \rho dA.$$

Green's theorem now tells us that we're looking for functions M and N such that $N_x - M_y = \rho x^2 + \rho y^2$. The simplest choice is $N_x = \rho y^2$, $M_y = -\rho x^2$. This leads to $N = \rho xy^2$, $M = -\rho x^2 y$.

Use Green's theorem to check this answer:

$$\begin{aligned} \oint_C -\rho x^2 y dx + \rho xy^2 dy &= \iint_R \rho y^2 - (-\rho x^2) dA \\ &= \iint_R r^2 \cdot \rho dA = I. \end{aligned}$$

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