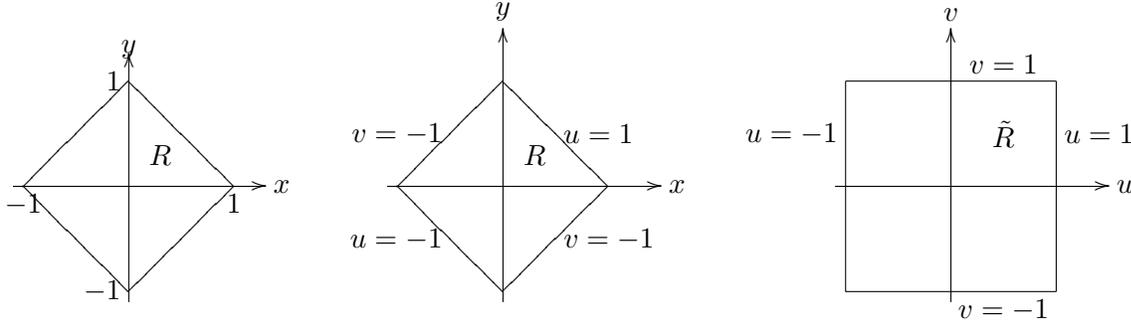


Problems: Change of Variables

Compute $\iint_R \left(\frac{x+y}{2-x+y} \right)^4 dx dy$, where R is the square with vertices at $(1,0)$, $(0,1)$, $(-1,0)$ and $(0,-1)$.

Answer: Since the region is bounded by the lines $x+y = \pm 1$ and $x-y = \pm 1$, we make a change of variables:

$$u = x + y \quad v = x - y.$$



Computing the Jacobian: $\frac{\partial(u,v)}{\partial(x,y)} = -2 \Rightarrow \frac{\partial(x,y)}{\partial(u,v)} = -1/2$.

Thus, $dx dy = \frac{1}{2} du dv$.

Using either method 1 or method 2 we see the boundaries are given by $u = \pm 1, v = \pm 1 \Rightarrow$ the integral is $\iint_R \left(\frac{x+y}{2-x+y} \right)^4 dx dy = \int_{-1}^1 \int_{-1}^1 \left(\frac{u}{2-v} \right)^4 \frac{1}{2} du dv$.

$$\text{Inner integral} = \frac{u^5}{10(2-v)^4} \Big|_{u=-1}^{u=1} = \frac{1}{5(2-v)^4}.$$

$$\text{Outer integral} = \frac{1}{15(2-v)^3} \Big|_{-1}^1 = \frac{26}{405} \approx .06.$$

We're integrating the fourth power of a fraction whose numerator ranges between -1 and 1 and whose denominator ranges between 1 and 3 . The value of this integrand will always be positive and will often be small, so this answer seems reasonable.

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18.02SC Multivariable Calculus
Fall 2010

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