Problems: Exchanging the Order of Integration

Calculate $\int_0^2 \int_x^2 e^{-y^2}\, dy\, dx$.

Answer: As you may recall, the function e^{-y^2} has no simple antiderivative. However, this double integral can be computed by reversing the order of integration.

The region R is the triangle with vertices at $(0,0)$, $(0,2)$ and $(2,2)$ (sketch it!) Thus:

$$\int_{x=0}^2 \int_{y=x}^2 e^{-y^2}\, dy\, dx = \int_{y=0}^2 \int_{x=0}^y e^{-y^2}\, dx\, dy.$$

Inner: $\int_{x=0}^y e^{-y^2}\, dx = ye^{-y^2}$.

Outer: $\int_{y=0}^2 ye{-}y^2\, dy = -\frac{1}{2}e^{-y^2}\Big|_0^2 = \frac{1}{2}(1 - e^{-4}) \approx \frac{1}{2}$.

We're finding the area under a surface with maximum height 1 and minimum height $e^{-4} \approx 0.1$ over a triangle of area 2. This answer seems plausible.

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