Tangent Plane to a Level Surface

1. Find the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 36$ at the point $P = (1, 2, 3)$.

Answer: In order to use gradients we introduce a new variable

$$w = x^2 + 2y^2 + 3z^2.$$

Our surface is then the the level surface $w = 36$. Therefore the normal to surface is

$$\boldsymbol{\nabla} w = \langle 2x, 4y, 6z \rangle.$$

At the point P we have $\boldsymbol{\nabla} w|_P = \langle 2, 8, 18 \rangle$. Using point normal form, the equation of the tangent plane is

$$2(x-1) + 8(y-2) + 18(z-3) = 0, \text{ or equivalently } 2x + 8y + 18z = 72.$$

2. Use gradients and level surfaces to find the normal to the tangent plane of the graph of $z = f(x, y)$ at $P = (x_0, y_0, z_0)$.

Answer: Introduce the new variable

$$w = f(x, y) - z.$$

The graph of $z = f(x, y)$ is just the level surface $w = 0$. We compute the normal to the surface to be

$$\boldsymbol{\nabla} w = \langle f_x, f_y, -1 \rangle.$$

At the the point P the normal is $\langle f_x(x_0, y_0), f_y(x_0, y_0), -1 \rangle$, so the equation of the tangent plane is

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0.$$

We can write this in a more compact form as

$$(z - z_0) = \left.\frac{\partial f}{\partial x}\right|_0 (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_0 (y - y_0),$$

which is exactly the formula we saw earlier for the tangent plane to a graph.

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