Chain Rule

1. The temperature on a hot surface is given by

$$T = 100\,\mathrm{e}^{-(x^2+y^2)}.$$

A bug follows the trajectory $\mathbf{r}(t) = \langle t\cos(2t), t\sin(2t)\rangle$.

a) What is the rate that temperature is changing as the bug moves?

b) Draw the level curves of T and sketch the bug's trajectory.

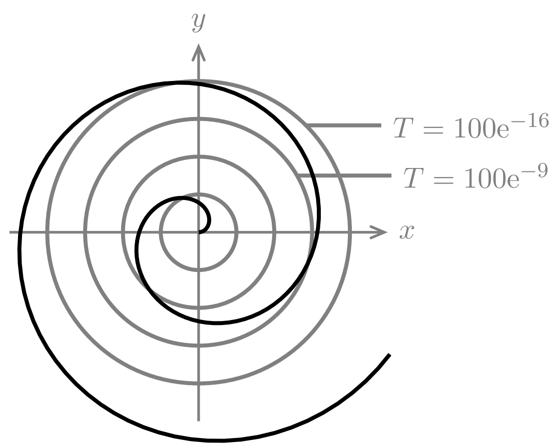
Answer: a) The chain rule says

$$\begin{aligned}\frac{dT}{dt} &= \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} \\ &= -200x\mathrm{e}^{-(x^2+y^2)}(\cos(2t) - 2t\sin(2t)) - 200y\mathrm{e}^{-(x^2+y^2)}(\sin(2t) + 2t\cos(2t)).\end{aligned}$$

You could stop here, or substitute $x = t\cos(2t)$ and $y = t\sin(2t)$. After simplification you get

$$\frac{dT}{dt} = -200\,t\,\mathrm{e}^{-t^2}.$$

b) The level curves of T are the curves $x^2 + y^2 =$ constant, i.e., circles. The bug moves in a spiral.



2. Suppose $w = f(x, y)$ and $x = t^2$, $y = t^3$. Suppose also that at $(x, y) = (1, 1)$ we have $\frac{\partial w}{\partial x} = 3$ and $\frac{\partial w}{\partial y} = 1$. Compute $\frac{dw}{dt}$ at $t = 1$.

Answer: At $t = 1$ we have $(x, y) = (1, 1)$, $\left.\frac{dx}{dt}\right|_1 = 2$, $\left.\frac{dy}{dt}\right|_1 = 3$. Therefore the chain rule says

$$\left.\frac{dw}{dt}\right|_1 = \left.\frac{\partial f}{\partial x}\right|_{(1,1)} \left.\frac{dx}{dt}\right|_1 + \left.\frac{\partial f}{\partial y}\right|_{(1,1)} \left.\frac{dy}{dt}\right|_1 = 3(2) + 1(3) = 9.$$