

Equation of a plane

1. Find the equation of the plane containing the three points $P_1 = (1, 0, 1)$, $P_2 = (0, 1, 1)$, $P_3 = (1, 1, 0)$.

Answer: This problem is identical (with changed numbers) to the worked example we just saw.

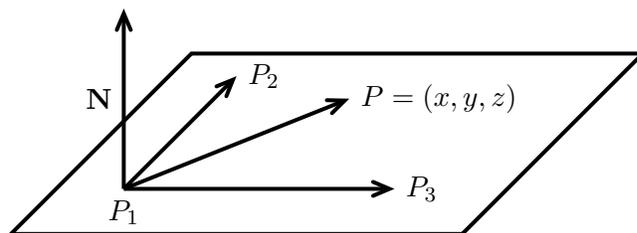
The vectors $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$ are in the plane, so

$$\mathbf{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \mathbf{i}(-1) - \mathbf{j}(1) + \mathbf{k}(-1) = \langle -1, -1, -1 \rangle.$$

is orthogonal to the plane.

Now for any point $P = (x, y, z)$ in the plane, the vector $\overrightarrow{P_1P}$ is also in the plane and is therefore orthogonal to \mathbf{N} . Expressing this with the dot product we get

$$\begin{aligned} \mathbf{N} \cdot \overrightarrow{P_1P} &= 0 \\ \Leftrightarrow \langle -1, -1, -1 \rangle \cdot \langle x-1, y, z-1 \rangle &= 0 \\ \Leftrightarrow -(x-1) - y - (z-1) &= 0 \\ \Leftrightarrow x + y + z &= 2. \end{aligned}$$



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