18.02 EXAM 4 REVIEW

1. Triple integrals; surface integrals (dV and dS)

Exercise. In each of rectangular, cylindrical and spherical coordinates, find the coordinate description of the following surfaces. Also find dS and $d\boldsymbol{S}$ for these surfaces. (This extra exercise is provided not because this topic is emphasized over the others, but rather because we studied these coordinate systems a while ago. Recall that equations for surfaces are used as limits of integration in triple integrals as well as in setting up surface integrals.)

a) the sphere around the origin ($\rho = a$); b) planes $x = a$, $y = a$, $z = a$; c) the cylinder $r = a$; d) the cone $\phi = \phi_0$; e) the sphere with origin at the South Pole ($\rho = 2a\cos\phi$).

Example. $x = a \iff r\cos\theta = a \iff r = a/\cos\theta$. If $0 \le a < b$, then the description of the "vertical slab" $a < x < b$ in cylindrical coodinates is

$$a/\cos\theta < r < b/\cos\theta; \quad -\pi/2 < \theta < \pi/2; \quad -\infty < z < \infty$$

(The limits on θ come from $\cos\theta > 0$.)

Evaluation of integrals. You will be provided with the usual table of powers of sine and cosine. Know how to integrate using substitution, such as the substitution $u = \sin\theta$ to integrate $\sin^n\theta\cos\theta\, d\theta$.

Types of integrals. You are expected to know the formula for mass and moments of inertia, and average value. Questions on Exam 4 in probability, if any, are limited to the all-important average value and probability as a ratio of masses, areas, or volumes, as in

$$\text{Probability} = \frac{\text{mass(part)}}{\text{mass(whole)}}$$

2. Line integrals in 3-D, gradient fields, curl, finding potential functions.

Test whether $\boldsymbol{F}$ is a gradient field by computing curl $\boldsymbol{F}$. Use a systematic method (your choice) to find a potential function. Use that potential function to compute line integrals (Fundamental theorem of calculus for line integrals).

3. Stokes' theorem. If the curve C is the boundary of the surface S and they are compatibly oriented then

$$\oint_C \boldsymbol{F}\cdot d\boldsymbol{r} = \iint_S (\nabla\times\boldsymbol{F})\cdot d\boldsymbol{S}$$

Key formulas: On a surface $g(x, y, z) = c$, $d\boldsymbol{S} = \pm\frac{\nabla g}{g_z}dxdy$. (Or, for example using y and z as coordinates, $d\boldsymbol{S} = \pm\frac{\nabla g}{g_x}dydz$.)

For graphs $z = f(x, y)$, $d\boldsymbol{S} = \pm\langle -f_x, -f_y, 1\rangle dxdy$. (Or, for example, if $x = h(y, z)$, then $d\boldsymbol{S} = \pm\langle 1, -h_y, -h_z\rangle dydz$.)

4. Divergence theorem. If the surface S encloses the solid region D, then

$$\iint_S \boldsymbol{F}\cdot d\boldsymbol{S} = \iiint_D \nabla\cdot\boldsymbol{F}\, dV$$

with $d\boldsymbol{S} = \boldsymbol{n}\, dS$ oriented so that $\boldsymbol{n}$ points away from D.