18.02 Practice Exam 4A

Problem 1.

Let R be the solid region defined by the inequalities

$$x^2 + y^2 + z^2 \leq a^2, \quad x \geq 0, \quad y \geq 0$$

(a) *(15)* Set up a triple integral in **cylindrical** coordinates which gives the volume of R. (Put in integrand and limits, but DO NOT EVALUATE.)

(b) *(15)* Find the formula in **spherical** coordinates which gives the average distance of points of R to the xz-plane.
(Put in integrand and limits, but DO NOT EVALUATE.)

Problem 2.

Let $\overrightarrow{\mathbf{F}}$ be the vector field $\langle axz, -1 - bz^2, x^2 - 2yz + 4\rangle$.

(a) *(10)* For what values of a and b will $\overrightarrow{\mathbf{F}}$ be a conservative field?

(b) *(10)* For these values of a and b find a potential function f for $\overrightarrow{\mathbf{F}}$.
Use a systematic method and show your work.

Problem 3.

Let $\overrightarrow{\mathbf{F}} = \langle xz, yz + x, xy\rangle$.

(a) *(10)* Find $\overrightarrow{\nabla} \times \overrightarrow{\mathbf{F}}$.

(b) *(15)* Let C be the simple closed curve (oriented counterclockwise when viewed from above) $x - y + 2z = 10$ whose projection onto the xy-plane is the circle $(x-1)^2 + y^2 = 1$.
By using Stokes' theorem, compute $\oint_C \overrightarrow{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

Problem 4.

(a) *(20)* Use the divergence theorem to compute the flux of $\overrightarrow{\mathbf{F}} = (1 + y^2)\hat{\jmath}$ out of the curved part of the half-cylinder bounded by $x^2 + y^2 = a^2$ $(y \geq 0)$, $z = 0$, $z = b$, and $y = 0$. **Justify your answer.**

(b) *(5)* Suppose that S is a closed surface that lies entirely in $y < 0$.
Is the **outward** flux of $\overrightarrow{\mathbf{F}} = (1 + y^2)\hat{\jmath}$ through S positive, negative, or zero? **Justify your answer.**