18.02: Practice Exam 3A

1. Let $(\bar{x}, \bar{y})$ be the center of mass of the triangle, with vertices at $(-2, 0)$, $(0, 1)$, $(2, 0)$ and uniform density $\delta = 1$.

a) Write an integral formula for $\bar{y}$. Do not evaluate the integral(s), but write explicitly the integrand and limits of integration.

b) Find $\bar{x}$.

2. Find the polar moment of inertia of the unit disk with density equal to the distance from the y-axis.

3. Let $\mathbf{F} = (ax^2y + y^3 + 1)\mathbf{i} + (2x^3 + bxy^2 + 2)\mathbf{j}$ be a vector field, where a and b are constants.

a) Find the values of a and b for which $\mathbf{F}$ is conservative.

b) For these values of a and b, find $f(x, y)$ such that $\mathbf{F} = \nabla f$.

c) Still using the values of a and b from part (a), compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C such that $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le \pi$.

4. For $\mathbf{F} = yx^3\mathbf{i} + y^2\mathbf{j}$ find $\int_C \mathbf{F} \cdot d\mathbf{r}$ on the portion of the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.

5. Consider the region R in the first quadrant bounded by the curves $y = x^2$, $y = x^2/5$, $xy = 2$, and $xy = 4$.

a) Compute $dxdy$ in terms of $dudv$ if $u = x^2/y$ and $v = xy$.

b) Find a double integral for the area of R in uv coordinates and evaluate it.

6. a) Let C be a simple closed curve going counterclockwise around a region R. Let $M = M(x, y)$. Express $\displaystyle\oint_C M dx$ as a double integral over R.

b) Find M so that $\displaystyle\oint_C M dx$ is the mass of R with density $\delta(x, y) = (x + y)^2$.

7. Consider the region R enclosed by the x-axis, $x = 1$ and $y = x^3$. Travelling in a counterclockwise direction along the boundary C or R, call C_1 the portion of C that goes from $(0, 0)$ to $(0, 1)$, C_2 the portion that goes from $(1, 0)$ to $(1, 1)$ and C_3 the portion that goes from $(1, 1)$ to $(0, 0)$.

a) Find the total work of $\mathbf{F} = (1 + y^2)\mathbf{i}$ around the boundary C of R, in a counterclockwise direction.

b) Calculate the work of $\mathbf{F}$ along C_1 and C_2.

c) Use parts (a) and (b) to find the work along the third side C_3.