

18.02 Problem Set 9 — Spring 2006

Due Thursday 4/20/06, 12:55 pm

Part A (5 points)

Hand in the underlined problems only; the others are for more practice.

Lecture 25. Thu Apr 13 Green's theorem.

Read: 21.3, Notes V4.3.

Work: 4D/ 1abc, 2, 3, 4, 5.

Lecture 26. Fri Apr 14 Simply-connected regions. Review.

Read: Notes V5, V6

Lecture 27. Thu Apr 20 **Exam 3.** (covering lectures 17–26, except triple integrals)

Part B (17 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Problem 0. (not until due date; 3 points)

Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say “none” or “no consultation”.

Problem 1. (Thursday; 4 points: 2 + 2)

a) Show that, if a simple, closed (positively oriented) curve C is the boundary of a region R , then $\text{Area}(R) = \oint_C x \, dy = \oint_C -y \, dx$.

b) Use this result to calculate the area of the region between the x -axis and one arch of the cycloid with parametric equations $x = a(t - \sin t)$, $y = a(1 - \cos t)$. (Hint: the second line integral leads to easier calculations.)

Problem 2. (Thursday; 5 points: 3 + 2)

a) For what simple, closed (positively oriented) curve C in the plane does the line integral $\oint_C (x^2y + y^3 - y) \, dx + (3x + 2y^2x + e^y) \, dy$ have the largest positive value? (Use Green's theorem.)

b) Determine what this value is.

Problem 3. (Thursday; 5 points: 3 + 2) Confirm Green's theorem for the line integral of $\mathbf{F} = y^2\mathbf{i} + x^2y\mathbf{j}$ around the loops from PS8, Part B Problem 1.

a) C_1 is the ellipse $x^2 + 4y^2 = 4$ traced counterclockwise (= positively oriented).

b) C_2 is the triangle in the first quadrant formed by the axes and the line $x + y = a$ traced counterclockwise.