18.02 Problem Set 2

Due Thursday 2/23/06, 12:55 pm

Part A (15 points)

Hand in the underlined problems only; the others are for more practice.

Lecture 5. Thu Feb. 16 Parametric curves, velocity, acceleration
Read: 18.4, 17.1, 17.4 Work: 1E/ 3abc, 4, 5; 1I/ 1, 3abd, 5.

Lecture 6. Fri Feb. 17 Kepler's second law
Read: 17.7, Notes K.
1J/ 1ac, 3, 4abc, 5, 6, 9, 10; 1K/ 2, 3.

(no lecture Tue Feb. 21 — Monday schedule)

Lecture 7. Thu Feb. 23 Exam 1 [Do not hand in the Practice Exam on back of this sheet. Solutions are available in the assignments section.]

Part B (18 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Problem 0. (not until due date; 2 points) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". (See full explanation on PS1).

Problem 1. (Thursday 7 points: 2+2+1+2)

a) Find the position vector of the trajectory of circular motion in the plane around the origin starting at $(-1, 0)$ going clockwise at unit speed.

b) Find the position vector of the trajectory of circular motion in the plane around the origin starting at $(10, 0)$ going counterclockwise at speed 60.

c) Do part (b) again but this time at speed 60 rpm (t measured in minutes).

d) Find the position and velocity vectors of the trajectory with initial position (at time $t = 0$) $\boldsymbol{r}_0 = \hat{\boldsymbol{j}}$, initial velocity $\boldsymbol{v}_0 = -\hat{\boldsymbol{i}}$, and acceleration $\boldsymbol{a} = (\cos t)\hat{\boldsymbol{i}} - (\sin t)\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$.

Problem 2. (Friday 9 points: 2 + 2 + 2 + 3)

a) Show that if a trajectory is on a sphere centered at the origin, then its velocity is tangent to the sphere. (Write the equation for the distance squared to the origin and differentiate.)

b) Show that the trajectory with position vector $\boldsymbol{r} = \cos t \sin(2t)\hat{\boldsymbol{i}} + \sin t \sin(2t)\hat{\boldsymbol{j}} + \cos(2t)\hat{\boldsymbol{k}}$ is on the unit sphere centered at the origin.

c) Find the velocity vector of the trajectory in part (b).

d) Find the angle this trajectory makes with the equator (intersection of the sphere with $z = 0$).