

18.02 Problem Set 10 Spring 2006

Due Thursday 4/27/06, 12:55 pm

Part A (15 points)

Hand in the underlined problems only; the others are for more practice.

Lecture 28. Fri Apr 21 Flux, normal form of Green's theorem.

Read: Notes V3, V4.

Work: 4E/ 1ac, 2, 3, 4, 5; 4F/4.

(In 4E/4, orienting C clockwise means the normal \hat{n} points into the square. In 4E/5, don't use Green's theorem.)

Lecture 29. Tue Apr 25 Vector fields in 3D; surface integrals and flux.

Read: Notes V8, V9.

Work: 6A/ 1, 2, 3, 4; 6B/ 1, 2, 3, 4, 6, 8.

(There is a misprint in Notes V9: the formula on page 5 should have the label (13') not (13), and likewise in the answer to 6B/6.)

Part B (22 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Problem 0. (not until due date; 4 points)

Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation".

Problem 1. (Friday, 9 points: 3 + 1 + 3 + 2) Here is a proof of the formulas in 3B using Green's theorem.

a) Define $A_n = \int_0^{2\pi} \sin^{2n} \theta d\theta$. Prove the recurrence formula

$$A_n = \frac{2n-1}{2n} A_{n-1} \quad (*)$$

by writing Green's theorem in normal form for the vector field $\mathbf{F} = y^{2n-1} \mathbf{j}$ in the unit disk $x^2 + y^2 < 1$ and evaluating each of the integrals separately. (Use polar coordinates, but do not use the formula at the top of 3B.)

b) Compute A_0 and derive, using (*), the formula for A_n . (Compare with 3B to make sure you have it right. Remember that the integrals there are from 0 to $\pi/2$, not 2π .)

c) Find and prove a recurrence formula similar to (*) for $B_n = \int_0^\pi \sin^{2n+1} \theta d\theta$ using the half-disk $y > 0$, $x^2 + y^2 < 1$ and the vector field $\mathbf{G} = y^{2n} \mathbf{j}$.

d) Compute B_0 and use the recurrence in (c) to find the formula for B_n . We used the half disk instead of the disk in part (c). It would have been easier to derive the recurrence for $\int_0^{2\pi} \sin^{2n+1} \theta d\theta$ using the whole disk. But this is useless. Why?

Problem 2. (Tuesday; 4 points: 2 + 2)

a) What percentage, rounded to the nearest percent, of the Earth's surface is south of Rio de Janeiro? The latitude there is 23 S, that is, 23° south of the Equator.

b) Find the average latitude of all points in the Northern Hemisphere. Our latitude in Boston is 42° . Are we above or below average? Optional: Identify a city whose latitude is within one degree of the average.

Problem 3. (Tuesday; 5 points) Compute the flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ out of the cylinder $x^2 + y^2 < a^2$, $0 \leq z \leq 1$ directly from the definition of flux. (Do not use the divergence theorem, which we will learn soon.)