 Let's try to discuss a bit how things relate to physics. There are two main things I want to discuss. One of them is what curl says about force fields and, in particular,a nice consequence of that concerning gravitational attraction. More about curl. If we have a velocity field, then we have seen that the curl measures the rotation affects. More precisely curl v measures twice the angular velocity, or maybe I should say the angular velocity vector because it also includes the axis of rotation. I should say maybe for the rotation part of a motion. For example, just to remind you, I mean we have seen this guy a couple of times, but if I give you a uniform rotation motion about the z, axes. That is a vector field in which the trajectories are going to be circles centered in the z-axis and our vector field is just going to be tangent to each of these circles. And, if you look at it from above, then you will have this rotation vector field that we have seen many times. Typically, the velocity vector for this would be minus yi plus yj times maybe a number that represents how fast we are spinning, the angular velocity in gradients per second. And then. if you compute the curl of this, you will end up with two omega times k. Now, the other kinds of vector fields we have seen physically are force fields. The question is what does the curl of a force field mean? What can we say about that? The interpretation is a little bit less obvious, but let's try to get some idea of what it might be. I want to remind you that if we have a solid in a force field, we can measure the torque exerted by the force on the solid. Maybe first I should remind you about what torque is in space. Let's say that I have a piece of solid with a mass, delta m for example, and I have a force that is being exerted to it. Let's say that maybe my force might be F times delta m. If you think, for example, a gravitational field. The gravitational force is actually the gravitational field times the mass. I mean you can forget delta m if you don't like it. And let's say that the position vector, which should be aiming for the origin, R is here. And now let's say that maybe this guy is at the end of some arm or some metal thing and I want to hold it in place. The force is going to exert a torque relative to the origin that will try to measure how much I am trying to swing this guy around the origin. And, consequently, how much effort I have to exert if I want to actually maintain its place by just holding it at the end of the stick here. So the torque is now a vector, which is just the cross-product of a position vector with a force. What the torque measures again is the rotation effects of the force. And if you remember the principle that the derivative of velocity, which is acceleration, is force divided by mass then the derivative of angular velocity should be angular acceleration which is related to the torque per unit mass. To just remind you, if I look at translation motions, say I am just looking at the point mass so there are no rotation effects then force divided by mass is acceleration, which is the derivative of velocity. And so what I am claiming is that for rotation effects we have a similar law, which maybe you have seen in 8.01. Well, it is one of the important things of solid mechanics, which is the torque of a force divided by the moment of inertia. I am cheating a little bit here. If you can see how I am cheating then I am sure you know how to state it correctly. And if you don't see how I am cheating then let's just ignore the details. [LAUGHTER] Is angular acceleration. And angular acceleration is the derivative of angular velocity. If I think of curl as an operation, which from a velocity field gives the angular velocity of its rotation effects, then you see that the curl of an acceleration field gives the angular acceleration in the rotation part of the acceleration effects. And, therefore, the curl of a force field measures the torque per unit moment of inertia. It measures how much torque its force field exerts on a small test solid placed in it. If you have a small solid somewhere, the curl will just measure how much your solid starts spinning if you leave it in this force field. In particular, a force field with no curl is a force field that does not generate any rotation motion. That means if you put an object in there that is completely immobile and you leave it in that force field, well, of course it might accelerate in some direction but it won't start spinning. While, if you put it in there spinning already in some direction, it should continue to spin in the same way. Of course, maybe there will be friction and things like that which will slow it down but this force field is not responsible for it. The cool consequence of this is if a force field F derives from a potential -- That is what we have seen about conservative forces. Our main concern so far has been to say if we have a conservative force field it means that the work of a force is the change in the energy. And, in particular, we cannot get energy for free out of it. And the change in the potential energy is going to be the change in kinetic energy. You have conservation of energy principles. There is another thing that we know now because if a force derives from a potential then that means its curl is zero. That is the criterion we have seen for a vector field to derive from a potential. And if the curl is zero then it means that this force does not generate any rotation effects. For example, if you try to understand where the earth comes from, well, the earth is spinning on itself as it goes around the sun. And you might wonder where that comes from. Is that causes by gravitational attraction? And the answer is no. Gravitational attraction in itself cannot cause the earth to start spinning faster or slower, at least if you assume the earth to be a solid, which actually is false. I mean basically the reason why the earth is spinning is because it was formed spinning. It didn't start spinning because of gravitational effects. And that is a rather deep purely mathematical consequence of understanding gravitation in this way. It is quite spectacular that just by abstract thinking we got there. What is the truth? Well, the truth is the earth, the moon and everything is slightly deformable. And so there is deformation, friction effects, tidal effects and so on. And these actually cause rotations to get slightly synchronized with each other. For example, if you want to explain why the moon is always showing the same face to the earth, why the rotation of a moon on itself is synchronized with its revolution around the earth, which is actually explained by friction effects over time and the gravitational attraction of the earth and the moon. There is something there, but if you took perfectly rigid, solid bodies then gravitation would never cause any rotation effects. Of course that tells us that we do not know how to answer the question of why is the earth spinning. That will be left for another physics class. I don't have a good answer to that. That was kind of 8.01-ish. Let me now move forward to 8.02 stuff. I want to tell you things about electric and magnetic fields. And, in fact, something that is known as Maxwell's equations. Just a quick poll.

How many of you have been taking 8.02 or something like that? OK. That is not very many. For most of you this is a preview. If you have been taking 8.02, have you seen Maxwell's equations, at least part of them? Yeah. OK. Then I am sure, in that case, you know better than me what I am going to talk about because I am not a physicist. But just in case. Maxwell's equations govern how electric and magnetic fields behave, how they are caused by electric charges and their motions. And, in particular, they explain a lot of things such as how electric devices work, but also how electromagnetic waves propagate. In particular, that explains light and all sorts of waves. It is thanks to them, you know, your cell phone, laptops and things like that work. Anyway. Hopefully most of you know that the electric field is a vector field that basically tells you what kind of force will be exerted on a charged particle that you put in it. If you have a particle carrying an electric charge then this vector field will tell you, basically there will be an electric force which is the charge times E that will be exerted on that particle. And that is what is responsible, for example, for the flow of electrons when you have a voltage difference. Because classically this guy is a gradient of a potential. And that potential is just electric voltage. The magnetic field is a little bit harder to think about if you have never seen it in physics, but it is what is causing, for example, magnets to work. Well, basically it is a force that is also expressed in terms of a vector field usually called B. Some people call it H but I am going to use B. And that force tends to cause it, if you have a moving charged particle, to deflect its trajectory and start rotating in a magnetic field. What it does is not quite as easy as what an electric field does. Just to give you formulas, the force caused by the electric field is the charge times the electric field. And the force caused by the magnetic field, I am never sure about the sign. Is that the correct sign? Good. Now, the question is we need to understand how these fields themselves are caused by the charged particles that are placed in them. There are various laws in there that explain what is going on. Let me focus today on the electric field. Maxwell's equations actually tell you about div and curl of these fields. Let's look at div and curl of the electric field. The first equation is called the Gauss-Coulomb law. And it says that the divergence of the electric field is equal to, so this is a just a physical constant, and what it is equal to depends on what units you are using. And this guy rho, well, it is not the same rho as in spherical coordinates because physicists somehow pretended they used that letter first. It is the electric charge density. It is the amount of electric charge per unit volume. What this tells you is that divergence of E is caused by the presence of electric charge. In particular, if you have an empty region of space or a region where nothing has electrical charge then E has divergence equal to zero. Now, that looks like a very abstract strange equation. I mean it is a partial differential equation satisfied by the electric field E. And that is not very intuitive in any way. What is actually more intuitive is what we get if we apply the divergence theorem to this equation. If I think now about any closed surface, and I want to think about the flux of the electric field out of that surface, we haven't really thought about what the flux of a force field does. And I don't want to get into that because there is no very easy answer in general, but I am going to explain soon how this can be useful sometimes. Let's say that we want to find the flux of the electric field out of a closed surface. Then, by the divergence theorem, that is equal to the triple integral of a region inside of div E dV, which is by the equation one over epsilon zero, that is this constant, times the triple integral of rho dV. But now, if I integrate the charge density over the entire region, then what I will get is actually the total amount of electric charge inside the region. That is the electric charge in D. This one tells us, in a more concrete way, how electric charges placed in here influence the electric field around them. In particular, one application of that is if you want to study capacitors. Capacitors are these things that store energy by basically you have two plates, one that contains positive charge and a negative charge. Then you have a voltage between these plates. And, basically, that can provide electrical energy to power maybe an electric circuit. That is not really a battery because it doesn't store energy in large enough amounts. But, for example, that is why when you switch your favorite gadget off it doesn't actually go off immediately but somehow you see things dimming progressively. There is a capacitor in there. If you want to understand how the voltage and the charge relate to each other, the voltage is obtained by integrating the electric field from one plate to the other plate. And the charges in the plates are what causes the electric field between the plates. That is how you can get the relation between voltage and charge in these guys. That is an example of application of that. Now, of course, if you haven't seen any of this then maybe it is a little bit esoteric, but that will tell you part of what you will see in 8.02. Questions? I see some confused faces. Well, don't worry. It will make sense some day. [LAUGHTER] The next one I want to tell you about is Faraday's law. In case you are confused, Maxwell's equations, there are four equations in the set of Maxwell's equations and most of them don't carry Maxwell's name. That is a quirky feature. That one tells you about the curl of the electric field. Now, depending on your knowledge, you might start telling me that the curl of the electric field has to be zero because it is the gradient of the electric potential. I told you this stuff about voltage. Well, that doesn't account for the fact that sometimes you can create voltage out of nowhere using magnetic fields. And, in fact, you have a failure of conservativity of the electric force if you have a magnetic field. What this one says is the curl of E is not zero but rather it is the derivative of the magnetic field with respect to time. More precisely it tells you that what you might have learned about electric fields deriving from electric potential becomes false if you have a variable magnetic field. And just to tell you again that is a strange partial differential equation relating these two vector fields. To make sense of it one should use Stokes' theorem. If we apply Stokes' theorem to compute the work done by the electric field around a closed curve, that means you have a wire in there and you want to find the voltage along the wire. Now there is a strange thing because classically you would say, well, if I just have a wire with nothing in it there is no voltage on it. Well, a small change in plans. If you actually have a varying magnetic field that passes through that wire then that will actually generate voltage in it. That is how a transformer works. When you plug your laptop into the wall circuit, you don't actually feed it directly 110 volts, 120 volts or whatever. There is a transformer in there. What the transformer does it takes some input voltage and passes that through basically a loop of wire. Not much seems to be happening. But now you have another loops of wire that is intertwined with it. Somehow the magnetic field generated by it, and it has to be a donating current. The donating current varies over time in the first wire. That generates a magnetic field that varies over time, so that causes 2B by 2t and that causes curl of the electric field.

And the curl of the electric field will generate voltage between these two guys. And that is how a transformer works. It uses Stokes' theorem. More precisely, how do we find the voltage between these two points? Well, let's close the loop and let's try to figure out the voltage inside this loop. To find a voltage along a closed curve places in a varying magnetic field, we have to do the line integral along a closed curve of the electric field. And you should think of this as the voltage generated in this circuit. That will be the flux for this surface bounded by the curve of curl E dot dS. That is what Stokes' theorem says. And now if you combine that with Faraday's law you end up with the flux trough S of minus dB over dt. And, of course, you could take, if your loop doesn't move over time, I mean there is a different story if you start somehow taking your wire and somehow moving it inside the field. But if you don't do that, if it is the field that is moving then you just can take the dB by dt outside. But let's not bother. Again, what this equation tells you is that if the magnetic field changes over time then it creates, just out of nowhere, and electric field. And that electric field can be used to power up things. I don't really claim that I have given you enough details to understand how they work, but basically these equations are the heart of understanding how things like capacitors and transformers work. And they also explain a lot of other things, but I will leave that to your physics teachers. Just for completeness, I will just give you the last two equations in that. I am not even going to try to explain them too much. One of them says that the divergence of the magnetic field is zero, which somehow is fortunate because otherwise you would run into trouble trying to understand surface independence when you apply Stokes' theorem in here. And the last one tells you how the curl of the magnetic field is caused by motion of charged particles. In fact, let's say that the curl of B is given by this kind of formula, well, J is what is called the vector of current density. It measures the flow of electrically charged particles. You get this guy when you start taking charged particles, like electrons maybe, and moving them around. And, of course, that is actually part of how transformers work because I have told you running the AC through the first loop generates a magnetic field. Well, how does it do that? It is thanks to this equation. If you have a current passing in the loop that causes a magnetic field and, in turn, for the other equation that causes an electric field, which in turn causes a current. It is all somehow intertwined in a very intricate way and is really remarkable how well that works in practice. I think that is basically all I wanted to say about 8.02. I don't want to put your physics teachers out of a job. [LAUGHTER] If you haven't seen any of this before, I understand that this is probably not detailed enough to be really understandable, but hopefully it will make you a bit curious about that and prompt you to take that class someday and maybe even remember how it relates to 18.02.