

Introduction to Limit Comparison

Recall that we're trying to find out whether the tail of a function is fat or thin — whether we can safely ignore the values of the function after a certain point or whether the area under the graph of the function is infinite. If you can't directly compute the area of the tail you can use *limit comparison* to compare your function to some function whose values you are able to compute.

If $f(x) \sim g(x)$ as x goes to infinity then $\int_a^\infty f(x) dx$ and $\int_a^\infty g(x) dx$ either both converge or both diverge.

(Remember that “ $f \sim g$ as x goes to infinity” means that $\frac{f(x)}{g(x)} \rightarrow 1$ as $x \rightarrow \infty$.)

Example:
$$\int_0^\infty \frac{dx}{\sqrt{x^2+10}}$$

Since $\sqrt{x^2+10} \sim \sqrt{x^2} = x$, we can compare this integral to $\int_1^\infty \frac{dx}{x}$. (Because $1/x$ is singular at $x=0$ we'll start by ignoring the finite value $\int_0^1 \frac{dx}{\sqrt{x^2+10}}$ — if it turns out that the integral converges we can add it later.)

$$\int_1^\infty \frac{dx}{\sqrt{x^2+10}} \sim \int_1^\infty \frac{dx}{x}$$

We know that $\int_1^\infty \frac{dx}{x}$ diverges, so limit comparison tells us that $\int_0^\infty \frac{dx}{\sqrt{x^2+10}}$ diverges as well.

Question: Why did we switch from 0 to 1?

Answer: Because $\int_0^1 \frac{dx}{x}$ is infinite for unrelated reasons, and we didn't want that to affect our results. The interval between 0 and 1 is not the part of the function that we care about. What we're really worried about is what the area of the tail of the function is. We could just as well have done the comparison:

$$\int_{100}^\infty \frac{dx}{\sqrt{x^2+10}} \sim \int_{100}^\infty \frac{dx}{x}$$

which leads us to the same conclusion — the area of the tail of the graph of $\frac{1}{\sqrt{x^2+10}}$ is infinite.

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