

**Example:**  $\int_1^\infty \frac{dx}{x}$

Our next examples of indefinite integrals come close to the dividing line between infinite integral values and finite ones. We're exploring this boundary because we often want to know how large a variable value we must account for before we can ignore the rest.

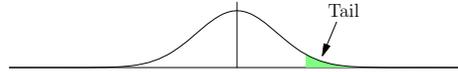


Figure 1: The tail of  $e^{-x^2}$ .

For example, when calculating probabilities the area of the “tail” of the normal curve gives the probability of an extreme value of  $x$  arising. If the area of the tail is negligible you don't have to compute it, but if it's not negligible (if it's a “fat tail”) and you don't take it into account you can get a nasty surprise — like the recent mortgage scandal. The job of a mathematician is to know what finite regions require careful calculation and what areas are too small to affect the final outcome.

How “fat” does a tail have to be before its area becomes infinite and overwhelms the area of the central body? The borderline cases are  $x^p$ , where  $p$  is negative. Let's start by looking at the case for which  $p = 1$ ; in other words:

$$\int_1^\infty \frac{dx}{x}.$$

As usual we start by computing the integral from 1 to  $N$  and then let  $N$  go to infinity.

$$\begin{aligned} \int_1^N \frac{dx}{x} &= \ln x \Big|_1^N \\ &= \ln N - \ln 1 \\ &= \ln N - 0 \end{aligned}$$

As  $N$  goes to infinity so does  $\int_1^N \frac{dx}{x}$ , so we conclude that:

$$\int_1^\infty \frac{dx}{x} \text{ diverges.}$$

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